Type: Article in Journal

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Title: Peirce Algebras and Boolean Modules

Year: 2007

Journal: Journal of Logic and Computation

Volume: 17

Issue: 2

Pages: 255-283

Keywords: Peirce algebra, Boolean module, Representation, Relation algebra, Axiomatisation

Abstract: A boolean module comprises a relation algebra, a boolean algebra and a Peircean operator which must obey a certain finite set of equational axioms. A Peirce algebra is a boolean module with one more operator, right cylindrification, which must obey another finite set of equational axioms. A Peirce algebra can be interpreted in a two-sorted algebra built within its relation algebra part, but no such interpretation can be found for boolean modules. We consider three different types of representations for relation algebras, boolean modules and Peirce algebras: classical (abbreviated as class.) representations, where all operators must be interpreted according to their natural set-theoretic definitions; non-additive (non-+) representations, where the relation algebra negation and sum need not be interpreted correctly; non-multiplicative (non-×) representations, where the relation algebra negation and intersection need not be interpreted correctly. For λ ∈ {class, non-+, non-×} the class of boolean modules with λ-representations and the class of Peirce algebras with λ-representations form quasi-varieties of two-sorted algebras. The class of classically representable boolean modules (respectively, Peirce algebras) is an equational variety, it is the variety generated by the class of all full boolean modules (Peirce algebras). A λ-representation of a boolean module or of a Peirce-algebra is straight if the representations of the boolean unit and the relation algebra unit have the same base set. For λ ∈ {class, non-+, non-×} a λ-representation of a Peirce algebra is necessarily straight, but there are λ-representable boolean modules with no straight λ-representations at all. For each such λ, write (i) RPAλ, (ii) RBMλ, (iii) RRAλ and (iv) SRBMλ for the classes consisting of Peirce-type algebras where (i) the algebra has a λ-representation, (ii) the reduct to boolean modules has a λ-representation, (iii) the reduct to

relation algebras has a λ-representation and (iv) the reduct to boolean modules has a straight λ-representation.

DOI: 10.1093/logcom/exl037

Language: English