‘Continuity’ (pub. 30.03.15-17:15). Quote in M. Bergman & S. Paavola (Eds.), *The Commens Dictionary: Peirce's Terms in His Own Words. New Edition*. Retrieved from http://www.commens.org/dictionary/entry/quote-law-mind-6.

Term: Continuity

Quote: We now come to the difficult question, What is continuity? Kant confounds it with infinite divisibility, saying that the essential character of a continuous series is that between any two members of it a third can always be found. This is an analysis beautifully clear and definite; but, unfortunately, it breaks down under the first test. For according to this, the entire series of rational fractions arranged in the order of their magnitude would be an infinite series, although the rational fractions are numerable, while the points of a line are innumerable. Nay, worse yet, if from that series of fractions any two with all that lie between them be excised, and any number of such finite gaps be made, Kant’s definition is still true of the series, though it has lost all appearance of continuity.

[—]

Kant’s definition expresses one simple property of a continuum; but it allows of gaps in the series. To mend the definition, it is only necessary to notice how these gaps can occur. Let us suppose, then, a linear series of points extending from a point, A, to a point, B, having a gap from B to a third point, C, and thence extending to a final limit, D; and let us suppose this series conforms to Kant’s definition. Then, of the two points, B and C, one or both must be excluded from the series; for otherwise, by the definition, there would be points between them. That is, if the series contains C, though it contains all the points up to B, it cannot contain B. What is required, therefore, is to state in non-metrical terms that if a series of points up to a limit is included in a continuum the limit is included. It may be remarked that this is the property of a continuum to which Aristotle’s attention seems to have been directed when he defines a continuum as something whose parts have a common limit. The property may be exactly stated as follows: If a linear series of points is continuous between two points, A and D, and if an endless series of points be taken, the first of them between A and D and each of the others between the last preceding one and D, then there is a point of the continuous series between all that endless series of points and D, and such that every other point of which this is true lies between this point and D. For example, take any number between 0 and 1, as 0.1; then, any number between 0.1 and 1, as 0.11; then any number between 0.11 and 1, as 0.111; and so on, without end. Then, because the series of real numbers between 0 and 1 is continuous, there must be a *least* real number, greater than every number of that endless series. This property, which may be called the Aristotelicity of the series, together with

Kant’s property, or its Kanticity, completes the definition of a continuous series.

The property of Aristotelicity may be roughly stated thus: a continuum contains the end point belonging to every endless series of points which it contains. An obvious corollary is that every continuum contains its limits. But in using this principle it is necessary to observe that a series may be continuous except in this, that it omits one or both of the limits.

Source: Peirce, C. S. (1892). The Law of Mind. *The Monist*, *2*(4), 533-559.

References: W 8:143-145; CP 6.120-123

Date of Quote: 1892

URL: http://www.commens.org/dictionary/entry/quote-law-mind-6