

'Continuity' (pub. 30.03.15-17:32). Quote in M. Bergman & S. Paavola (Eds.), *The Commens Dictionary: Peirce's Terms in His Own Words. New Edition*. Retrieved from <http://www.commens.org/dictionary/entry/quote-peirces-personal-interleaved-copy-century-dictionary-commens-0>.

Term: Continuity

Quote: ...I made a new definition, according to which continuity consists in *Kanticity* and *Aristotelicity*. The Kanticity is having a point between any two points. The Aristotelicity is having every point that is a limit to an infinite series of points that belong to the system.

I here slightly modify Cantor's definition of a perfect system. Namely, he defines it as such that it contains every point in the neighborhood of an infinity of points and no other. But the latter is a character of a concatenated system; hence I omit it as a character of a perfect system.

But further study of the subject has proved that this definition is wrong. It involves a misunderstanding of Kant's definition which *he himself* likewise fell into. Namely he defines a continuum as that all of whose parts have parts of the same kind. He himself, and I after him, understood that to mean infinite divisibility, which plainly is not what constitutes continuity since the series of rational fractional values is infinitely divisible but is not by anybody regarded as continuous. Kant's real definition implies that a continuous line contains no points. Now if we are to accept the common sense idea of continuity (after correcting its vagueness and fixing it to mean something) we must either say that a continuous line contains no points or we must say that the principle of excluded middle does not hold of these points. The principle of excluded middle only applies to an individual (for it is not true that "Any man is wise" nor that "Any man is not wise"). But places, being mere possibles without actual existence, are not individuals. Hence a point or indivisible place really does not exist unless there actually be something there to mark it, which, if there is, interrupts the continuity. I, therefore, think that Kant's definition correctly defines the common sense idea, although there are great difficulties with it. I certainly think that on any line whatever, on the common sense idea, there is room for any multitude of points however great. If so, the *analytical continuity* of the theory of functions, which implies there is but a single point for each distance from the origin, defined by a quantity expressible to indefinitely close approximation by a decimal carried out to an indefinitely great number of places, is certainly not the continuity of common sense, since the whole multitude of such quantities is only the first abnumeral multitude, and there is an infinite series of higher grades. On the whole, therefore, I think we must say that continuity is the relation of the parts of an unbroken space or time. The

precise definition is still in doubt; but Kant's definition, that a continuum is that of which every part has itself parts of the same kind, seems to be correct. This must not be confounded (as Kant himself confounded it) with infinite divisibility, but implies that a line, for example, contains no points until the continuity is broken by marking the points. In accordance with this it seems necessary to say that a continuum, where it is continuous and unbroken, contains no definite parts; that its parts are created in the act of defining them and the precise definition of them breaks the continuity. In the calculus and theory of functions it is assumed that between any two rational points (or points at distances along the line expressed by rational fractions) there are rational points and that further for every convergent series of such fractions (such as 3.1, 3.14, 3.141, 3.1415, 3.14159, etc.) there is just one limiting point; and such a collection of points is called *continuous*. But this does not seem to be the common sense idea of continuity. It is only a collection of independent points. Breaking grains of sand more and more will only make the sand more broken. It will not weld the grains into unbroken continuity.

Source: Peirce, C. S. (nd). *Peirce's Personal Interleaved Copy of the 'Century Dictionary' [Commens]*.

References: CP 6.166

Date of 1903

Quote:

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