All positive reasoning is of the nature of judging the proportion of something in a whole collection by the proportion found in a sample. Accordingly, there are three things to which we can never hope to attain by reasoning, namely, absolute certainty, absolute exactitude, absolute universality. We cannot be absolutely certain that our conclusions are even approximately true; for the sample may be utterly unlike the unsampled part of the collection. We cannot pretend to be even probably exact; because the sample consists of but a finite number of instances and only admits special values of the proportion sought. Finally, even if we could ascertain with absolute certainty and exactness that the ratio of sinful men to all men was as 1 to 1; still among the infinite generations of men there would be room for any finite number of sinless men without violating the proportion. The case is the same with a seven legged calf.

Now if exactitude, certitude, and universality are not to be attained by reasoning, there is certainly no other means by which they can be reached.

... let me call your attention to the natural affinity of this principle to the doctrine of fallibilism. The principle of continuity is the idea of fallibilism objectified. For fallibilism is the doctrine that our knowledge is never absolute but always swims, as it were, in a continuum of uncertainty and of indeterminacy. Now the doctrine of continuity is that all things so swim in continua.

... On the whole, then, we cannot in any way reach perfect certitude nor exactitude. We never can be absolutely sure of anything, nor can we with any probability ascertain the exact value of any measure or general ratio.

This is my conclusion, after many years study of the logic of science; and it is the conclusion which others, of very different cast of mind, have come to, likewise. I believe I may say there is no tenable opinion regarding human knowledge which does not legitimately lead to this corollary. Certainly there is nothing new in it; and many of the greatest minds of all time have held it for true.

[—] But it would be quite misunderstanding the doctrine of fallibilism to suppose that it means that twice two is probably not exactly four. As I have already remarked, it is not my purpose to doubt that people can usually count with accuracy. Nor does fallibilism say that men cannot attain a sure knowledge of the creations of their own minds. It neither affirms nor denies that. It only says that people cannot attain absolute certainty concerning questions of fact.
To return to our friends the Conservatives; these ladies and gentlemen will tell me this doctrine of fallibilism can never be admitted because the consequences from it would undermine Religion. I can only say I am very sorry. The doctrine is true; - without claiming absolute certainty for it, it is substantially unassailable. And if its consequences are antagonistic to religion, so much the worse for religion. At the same time, I do not believe they are so antagonistic. The dogmas of a church may be infallible – infallible in the sense in which it is infallibly true that it is wrong to murder and steal – practically and substantially infallible. But what use a church could make of a mathematical infallibility, I fail to see.

Thus, the universe is not a mere mechanical result of the operation of blind law. The most obvious of all its characters cannot be so explained. It is the multitudinous facts of all experience that show us this; but that which has opened our eyes to these facts is the principle of fallibilism. Those who fail to appreciate the importance of fallibilism reason: we see these laws of mechanics; we see how extremely closely they have been verified in some cases. We suppose that what we haven’t examined is like what we have examined, and that these laws are absolute, and the whole universe is a boundless machine working by the blind laws of mechanics. This is a philosophy which leaves no room for a God! No, indeed! It leaves even human consciousness, which cannot well be denied to exist, as a perfectly idle and functionless flâneur in the world, with no possible influence upon anything – not even upon itself. Now will you tell me that this fallibilism amounts to nothing?

For years [—] I used for myself to collect my ideas under the designation fallibilism; and indeed the first step toward finding out is to acknowledge you do not satisfactorily know already; so that no blight can so surely arrest all intellectual growth as the blight of cocksureness; and ninety-nine out of every hundred good heads are reduced to impotence by that malady – of whose inroads they are most strangely unaware!

Indeed, out of a contrite fallibilism, combined with a high faith in the reality of knowledge, and an intense desire to find things out, all my philosophy has always seemed to me to grow... .

No; but there is nothing at all in our knowledge which we have any warrant at all for regarding as absolute in any particular. Absolute infallibility may belong to the pope and the ecumenical councils: it is outside my province to discuss that question. But I am quite confident it does not belong to the multiplication table. If I must make any exception, let it be that the assertion that every assertion but this is fallible, is the only one that is absolutely infallible. But though nothing else is absolutely infallible, many propositions are practically infallible; such as the dicta of conscience. As for those
things which are known by everyday experience, let him doubt them who can lay his hand on his heart and say that he does doubt them. For the rest of us, it would be mendacity to say that our degree of assurance of them is unsatisfactory.

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The method of this book, therefore, is to accept the reasonings of pure mathematics as beyond all doubt. It is fallible, as everything human is fallible. Twice two may perhaps not be four.