Induction

1866 | Lowell Lectures on The Logic of Science; or Induction and Hypothesis: Lecture V | W 1:428

These differences between these two scientific inferences are so great that it seems to me essential to a right understanding of the subject that we should recognize two kinds of scientific reasoning, Induction and Hypothesis. Induction is the process by which we find the general characters of classes and establish natural classifications. [—] So that we have

Deduction
Induction
and Hypothesis

as three coördinate classes of reasoning.

1867 | On a New List of Categories | W 2:58; CP 1.559

In an argument, the premises form a representation of the conclusion, because they indicate the interpretant of the argument, or representation representing it to represent its object. The premises may afford a likeness, index, or symbol of the conclusion. In deductive argument, the conclusion is represented by the premises as by a general sign under which it is contained. In hypotheses, something *like* the conclusion is proved, that is, the premises form a likeness of the conclusion. [—] That it is different with induction another example will show.

SI, SII, SIII, and SIV are taken as samples of the collection M;
SI, SII, SIII, and SIV are P:
.·. All M is P.

Hence the first premise amounts to saying that “SI, SII, SIII, and SIV” is an index of M. Hence the premises are an index of the conclusion.

1867 | On the Natural Classification of Arguments | CP 2.515

Induction may, therefore, be defined as argument which assumes that a whole collection, from which a number of instances have been taken at random, has all the common characters of those instances …

1867 | On the Natural Classification of Arguments | W 2:46; CP 2.511

Hence the formulæ are

Induction
S′ S″ S‴′, &c. are taken at random as *M*'s,
S′ S″ S‴′, &c. are *P*;
.·. Any *M* is probably *P*.

1868 | Some Consequences of Four Incapacities | CP 5.275

Induction may be defined as an argument which proceeds upon the assumption that all the members of a class or aggregate have all the characters which are common to all those members of this class concerning which it is known, whether they have these characters or not; or, in other words, which assumes that that is true of a whole collection which is true of a number of instances taken from it at random. This might be called statistical argument. In the long run, it must generally afford pretty correct conclusions from true premisses. If we have a bag of beans partly black and partly white, by counting the relative proportions of the two colors in several different handfuls, we can approximate more or less to the relative proportions in the whole bag, since a sufficient number of handfuls would constitute all the beans in the bag. The central characteristic and key to induction is, that by taking the conclusion so reached as major premiss of a syllogism, and the proposition stating that such and such objects are taken from the class in question as the minor premiss, the other premiss of the induction will follow from them deductively. [—] Accordingly, induction has been defined by Aristotle as the inference of the major premiss of a syllogism from its minor premiss and conclusion. The function of an induction is to substitute for a series of many subjects, a single one which embraces them and an indefinite number of others. Thus it is a species of "reduction of the manifold to unity."

1878 | Deduction, Induction, and Hypothesis | CP 2.623

… So that induction is the inference of the *rule* from the *case* and *result*.
But this is not the only way of inverting a deductive syllogism so as to produce a synthetic inference. [—] We have, then–

DEDUCTION.

Rule.–All the beans from this bag are white.
Case.–These beans are from this bag.
.·.*Result*.–These beans are white.

INDUCTION.

Case.–These beans are from this bag.
Result.–These beans are white.
.·.*Rule*.–All the beans from this bag are white

HYPOTHESIS.

Rule.–All the beans from this bag are white.
Result.–These beans are white.
.·.*Case*.–These beans are from this bag.

We, accordingly, classify all inference as follows:

Inference.

|————————————|
Deductive or Analytic. Synthetic.
|————————-|
Induction. Hypothesis.

Induction is where we generalize from a number of cases of which something is true, and infer that the same thing is true of a whole class. Or, where we find a certain thing to be true of a certain proportion of cases and infer that it is true of the same proportion of the whole class.

1878 | Deduction, Induction, and Hypothesis | CP 2.636

… the distinction between induction and hypothesis. In the main, it is broad and decided. By induction, we conclude that facts, similar to observed facts, are true in cases not examined. By hypothesis, we conclude the existence of a fact quite different from anything observed, from which, according to known laws, something observed would necessarily result. The former, is reasoning from particulars to the general law; the latter, from effect to cause. The former classifies, the latter explains. It is only in some special cases that there can be more than a momentary doubt to which category a given inference belongs. One exception is where we observe, not facts similar under similar circumstances, but facts different under different circumstances–the difference of the former having, however, a definite relation to the difference of the latter. Such inferences, which are really inductions, sometimes present, nevertheless, some indubitable resemblances to hypotheses.

1878 | Deduction, Induction, and Hypothesis | CP 2.643; W 3:337

Induction infers a rule. Now, the belief of a rule is a habit. That a habit is a rule active in us, is evident. That every belief is of the nature of a habit, in so far as it is of a general character, has been shown in the earlier papers of this series. Induction, therefore, is the logical formula which expresses the physiological process of formation of a habit. [—] We may say, therefore, that hypothesis produces the *sensuous* element of thought, and induction the *habitual* element.

1883 | A Theory of Probable Inference | CP 2.702-703

The principle of statistical deduction is that these two proportions–namely, that of the *P*'s among the *M*'s, and that of the *P*'s among the *S*'s–are probably and approximately equal. If, then, this principle justifies our inferring the value of the second proportion from the known value of the first, it equally justifies our inferring the value of the first from that of the second, if the first is unknown but the second has been observed. We thus obtain the following form of inference:

FORM V

Induction.

S′, S″, S‴, etc. form a numerous set taken at random from among the *M*'s,
S′, S″, S‴, etc. are found to be–the proportion {r} of them–*P*'s;
Hence, *probably* and *approximately* the same proportion, {r}, of the *M*'s are *P*'s.

The following are examples. From a bag of coffee a handful is taken out, and found to have nine-tenths of the beans perfect; whence it is inferred that about nine-tenths of all the beans in the bag are probably perfect. [—]

When the ratio {r} is *unity* or *zero*, the inference is an ordinary induction; and I ask leave to extend the term "induction" to all such inference, whatever be the value of {r}. It is, in fact, inferring from a sample to the whole lot sampled. These two forms of inference, statistical deduction and induction, plainly depend upon the same principle of equality of ratios, so that their validity is the same. Yet the nature of the probability in the two cases is very different. In the statistical deduction, we know that among the whole body of *M*'s the proportion of *P*'s is {r}; we say, then, that the *S*'s being random drawings of *M*'s are probably *P*'s in about the same proportion–and though this may happen not to be so, yet at any rate, on continuing the drawing sufficiently, our prediction of the ratio will be vindicated at last. On the other hand, in induction we say that the proportion {r} of the sample being *P*'s, probably there is about the same proportion in the whole lot; or at least, if this happens not to be so, then on continuing the drawings the inference will be, not *vindicated* as in the other case, but *modified* so as to become true. The deduction, then, is probable in this sense, that though its conclusion may in a particular case be falsified, yet similar conclusions (with the same ratio {r}) would generally prove approximately true; while the induction is probable in this sense, that though it may happen to give a false conclusion, yet in most cases in which the same precept of inference was followed, a different and approximately true inference (with the right value of {r}) would be drawn.

1883 | A Theory of Probable Inference | CP 2.708-709

The following examples will illustrate the distinction between statistical deduction, induction, and hypothesis. If I wished to order a font of type expressly for the printing of this book, knowing, as I do, that in all English writing the letter e occurs oftener than any other letter, I should want more e's in my font than other letters. For what is true of all other English writing is no doubt true of these papers. This is a statistical deduction. But then the words used in logical writings are rather peculiar, and a good deal of use is made of single letters. I might, then, count the number of occurrences of the different letters upon a dozen or so pages of the manuscript, and thence conclude the relative amounts of the different kinds of type required in the font. That would be inductive inference. If now I were to order the font, and if, after some days, I were to receive a box containing a large number of little paper parcels of very different sizes, I should naturally infer that this was the font of types I had ordered; and this would be hypothetic inference. [—]

We are thus led to divide all probable reasoning into deductive and ampliative, and further to divide ampliative reasoning into induction and hypothesis. In deductive reasoning, though the predicted ratio may be wrong in a limited number of drawings, yet it will be approximately verified in a larger number. In ampliative reasoning the ratio may be wrong, because the inference is based on but a limited number of instances; but on enlarging the sample the ratio will be changed till it becomes approximately correct. In induction, the instances drawn at random are numerable things; in hypothesis they are characters, which are not capable of strict enumeration, but have to be

otherwise estimated.

1883 | A Theory of Probable Inference | CP 2.712-713

... Induction proceeds from Case and Result to Rule; it is the formula of the formation of a habit or general conception–a process which, psychologically as well as logically, depends on the repetition of instances or sensations. [—]

[—] Conceiving of nature in this way, we naturally conceive of science as having three tasks–(1) the discovery of Laws, which is accomplished by induction; (2) the discovery of Causes, which is accomplished by hypothetic inference; and (3) the predictio of Effects, which is accomplished by deduction. It appears to me to be highly useful to select a system of logic which shall preserve all these natural conceptions.

1883 | A Theory of Probable Inference | CP 2.715

We now come to the consideration of the Rules which have to be followed in order to make valid and strong Inductions and Hypotheses. These rules can all be reduced to a single one; namely, that the statistical deduction of which the Induction or Hypothesis is the inversion, must be valid and strong.

1892 | The Law of Mind | CP 6.144-147

The three main classes of logical inference are Deduction, Induction, and Hypothesis. These correspond to three chief modes of action of the human soul. [—]

By induction, a habit becomes established. Certain sensations, all involving one general idea, are followed each by the same reaction; and an association becomes established, whereby that general idea gets to be followed uniformly by that reaction. [—]

Thus, by induction, a number of sensations followed by one reaction become united under one general idea followed by the same reaction; while, by the hypothetic process, a number of reactions called for by one occasion get united in a general idea which is called out by the same occasion. By deduction, the habit fulfills its function of calling out certain reactions on certain occasions.

The inductive and hypothetic forms of inference are essentially probable inferences, not necessary; while deduction may be either necessary or probable.

1896 [c.] | Lessons of the History of Science | CP 1.67

Induction is that mode of reasoning which adopts a conclusion as approximate, because it results from a method of inference which must generally lead to the truth in the long run. For example, a ship enters port laden with coffee. I go aboard and sample the coffee. Perhaps I do not examine over a

hundred beans, but they have been taken from the middle, top, and bottom of bags in every part of the hold. I conclude by *induction* that the whole cargo has approximately the same value per bean as the hundred beans of my sample. All that induction can do is to ascertain the value of a ratio.

1900-05-20 | Smithsonian Institution letters | HP 2:876-877

In 1867, I produced what I considered, and still consider proof that all arguments are of three kinds Deduction, Induction and Hypothesis, with a supplementary kind Analogy sharing in the nature of Induction and of Hypothesis. In various publications, I gradually made my doctrine more definite, until in 1883 I gave an account of it in *Studies in Logic by Members of the Johns Hopkins University*. The theory there given seems to me substantially correct as far as Induction goes. Later, I was led to see objections to the method in which I there dealt with Hypothesis, in regard to which I had departed from my earlier opinions; and in order to meet these objections, I at first proposed slightly to modify my theory both of Induction and of Hypothesis, leaving my later opinions about their relations to one another, as they were. But this new view on further consideration was found not to be acceptable in regard to Induction; and finally some five years ago I made an entirely fresh investigation, more careful than ever, the result of which was that I return to my early views on the relations of induction and hypothesis, leave the theory of induction as I had it in 1883 substantially, and restrict the modifications of it to hypothesis only. I think I may be confident of having the matter right now. At any rate, several careful re-criticisms of it have not disclosed any faults.

1900-05-20 | Smithsonian Institution letters | HP 2:878

Induction consists in taking samples of a genus and observing how many fall into a certain species, and thence concluding the probable and approximate value of the probability that in that genus any given individual will belong to that species. It supposes that there is a certain course of experience, and that the sample has been so drawn as to be governed by that same course of experience. Induction ascertains a probability and nothing more. We can calculate mathematically and therefore deductively *precisely* how often an induction conforming to certain conditions will give the true probability to a given degree of accuracy supposing that true probability to be known; and though not precisely, yet within certain limits, how often such an induction will give the probability within certain limits of accuracy even if the true probability is not given. Still, that leaves us entirely in the dark as to the probability that the ascertained probability in any particular case is within any named limits correct. Indeed, it is doubtful whether any meaning can be attached to such a question. All we have to do is to accept the result of induction provisionally, with a degree of confidence depending on the probable accuracy of the proceeding, without troubling our heads about the probability of the inferred ratio; and go ahead to get new observations to confirm or modify that ratio.

1901 | On the Logic of Drawing History from Ancient Documents Especially from Testimonies (Logic of History) | CP 7.218

Abduction, on the other hand, is merely preparatory. It is the first step of scientific reasoning, as induction is the concluding step. Nothing has so much contributed to present chaotic or erroneous

ideas of the logic of science as failure to distinguish the essentially different characters of different elements of scientific reasoning; and one of the worst of these confusions, as well as one of the commonest, consists in regarding abduction and induction taken together (often mixed also with deduction) as a simple argument. Abduction and induction have, to be sure, this common feature, that both lead to the acceptance of a hypothesis because observed facts are such as would necessarily or probably result as consequences of that hypothesis. But for all that, they are the opposite poles of reason, the one the most ineffective, the other the most effective of arguments. The method of either is the very reverse of the other's. Abduction makes its start from the facts, without, at the outset, having any particular theory in view, though it is motived by the feeling that a theory is needed to explain the surprising facts. Induction makes its start from a hypothesis which seems to recommend itself, without at the outset having any particular facts in view, though it feels the need of facts to support the theory. Abduction seeks a theory. Induction seeks for facts. In abduction the consideration of the facts suggests the hypothesis. In induction the study of the hypothesis suggests the experiments which bring to light the very facts to which the hypothesis had pointed. The mode of suggestion by which, in abduction, the facts suggest the hypothesis is by *resemblance*, – the resemblance of the facts to the consequences of the hypothesis. The mode of suggestion by which in induction the hypothesis suggests the facts is by *contiguity*, – familiar knowledge that the conditions of the hypothesis can be realized in certain experimental ways.

1901 | On the Logic of Drawing History from Ancient Documents Especially from Testimonies (Logic of History) | CP 7.208-217

It is desirable to consider a large range of inductions, with a view to distinguishing accurately between induction and abduction, which have generally been much confounded. I will, therefore, mention that, in the present state of my studies, I think I recognize three distinct genera of induction. I somewhat hesitate to publish this division; but it might take more years than I have to live to render it as satisfactory as I could wish. [—]

The first genus of induction is where we judge what approximate proportion of the members of a collection have a predesignate character by a sample drawn under one or other of the following three conditions, forming three species of this genus. [—]

The second genus of induction comprises those cases in which the inductive method if persisted in will certainly in time correct any error that it may have led us into; but it will not do so gradually, inasmuch as it is not quantitative; – not but that it may relate to quantity, but it is not a quantitative induction. It does not discover a ratio of frequency. [—]

I seem to recognize a third genus of inductions where we draw a sample of an aggregate which can not be considered as a collection, since it does not consist of units capable of being either counted or measured, however roughly; and where probability therefore cannot enter; but where we can draw the distinction of much and little, so that we can conceive of measurement being established; and where we may expect that any error into which the sampling will lead us, though it may not be corrected by a mere enlargement of the sample, or even by drawing other similar samples, yet must be brought to light, and that gradually, by persistence in the same general method. [—]

[—] We have now passed in review all the logically distinct forms of pure induction. It has been seen that one and all are mere processes for testing hypotheses already in hand. The induction adds nothing. At the very most it corrects the value of a ratio or slightly modifies a hypothesis in a way

which had already been contemplated as possible.

1901 | On the Logic of Drawing History from Ancient Documents Especially from Testimonies (Logic of History) | CP 7.207

… the distinction in respect to the nature of their validity between deduction and induction consists in this, – namely, deduction professes to show that certain admitted facts could not exist, even in an ideal world constructed for the purpose, either without the existence of the very fact concluded, or without the occurrence of this fact in the long run in that proportion of cases of the fulfilment of certain objective conditions in which it is concluded that it will occur, or in other words, without its having the concluded objective probability. In either case, deductive reasoning is necessary reasoning, although, in the latter case, its subject matter is probability. Induction, on the other hand, is not justified by any relation between the facts stated in the premisses and the fact stated in the conclusion; and it does not infer that the latter fact is either necessary or objectively probable. But the justification of its conclusion is that that conclusion is reached by a method which, steadily persisted in, must lead to true knowledge in the long run of cases of its application, whether to the existing world or to any imaginable world whatsoever.

1901 | On the Logic of Drawing History from Ancient Documents Especially from Testimonies (Logic of History) | CP 7.206

Having, then, by means of deduction, drawn from a hypothesis predictions as to what the results of experiment will be, we proceed to test the hypothesis by making the experiments and comparing those predictions with the actual results of the experiment. Experiment is very expensive business, in money, in time, and in thought; so that it will be a saving of expense, to begin with that positive prediction from the hypothesis which seems least likely to be verified. For a single experiment may absolutely refute the most valuable of hypotheses, while a hypothesis must be a trifling one indeed if a single experiment could establish it. When, however, we find that prediction after prediction, notwithstanding a preference for putting the most unlikely ones to the test, is verified by experiment, whether without modification or with a merely quantitative modification, we begin to accord to the hypothesis a standing among scientific results. This sort of inference it is, from experiments testing predictions based on a hypothesis, that is alone properly entitled to be called *induction*.

1902 | Carnegie Institution Correspondence | NEM 4:38

Induction is the highest and most typical form of reasoning. In my essay of 1883, I only recognized two closely allied logical forms of pure induction, one of which is undoubtedly the highest. I have since discovered eight other forms which include those almost exclusively used by reasoners who are not adepts in logic.

1902 | Minute Logic: Chapter I. Intended Characters of this Treatise | CP 2.102

The discussion of probability naturally brings us to the interesting question of the validity of induction. I undertake to demonstrate mathematically that the validity of Induction, in the proper sense of the term, that is to say, experimental reasoning, follows, through the lemmas of probabilities, from the rudiments of the doctrine of necessary consequences, without any assumption whatever about the future being like the past, or similar results following similar conditions, or the uniformity of nature, or any such vague principle.

1902 | Minute Logic: Chapter I. Intended Characters of this Treatise | CP 2.96

Argument is of three kinds: *Deduction*, *Induction*, and *Abduction* (usually called adopting a hypothesis). A Transuasive Argument, or *Induction*, is an Argument which sets out from a hypothesis, resulting from a previous Abduction, and from virtual predictions, drawn by Deduction, of the results of possible experiments, and having performed the experiments, concludes that the hypothesis is true in the measure in which those predictions are verified, this conclusion, however, being held subject to probable modification to suit future experiments. Since the significance of the facts stated in the premisses depends upon their predictive character, which they could not have had if the conclusion had not been hypothetically entertained, they satisfy the definition of a Symbol of the fact stated in the conclusion. This argument is Transuasive, also, in respect to its alone affording us a reasonable assurance of an ampliation of our positive knowledge. By the term “virtual prediction,” I mean an experiential consequence deduced from the hypothesis, and selected from among possible consequences independently of whether it is known, or believed, to be true, or not; so that at the time it is selected as a test of the hypothesis, we are either ignorant of whether it will support or refute the hypothesis, or, at least, do not select a test which we should not have selected if we had been so ignorant.

1902 | Minute Logic: Chapter I. Intended Characters of this Treatise | MS [R] 425:120-121

Arguments are of three kinds, *Deduction*, *Induction*, and what I call *Abduction* [—] If the conclusion is accepted because, having been hypothetically propounded as a hypothesis, it predicted that experiments would yield results which they have thereupon been found to yield, the argument is Inductive.

From early/discarded draft

1903 | Harvard Lectures on Pragmatism: Lecture V, a deleted passage | PPM 276-277

Now, I said, Abduction, or the suggestion of an explanatory theory, is inference through an Icon, and is thus connected with Firstness; Induction, or trying how things will act, is inference through an Index, and is thus connected with Secondness; Deduction, or recognition of the relations of general ideas, is inference through a Symbol, and is thus connected with Thirdness. [—] But my connection of Abduction with Firstness, Induction with Secondness, and Deduction with Thirdness was confirmed by my finding

no essential subdivision of Abductions, that Induction split at once, into the Sampling of Collections, and the Sampling of Qualities, while in the logic of relatives the three figures of syllogism gain a reality which is not so easily perceived in non-relative syllogism but really exists there also.

1903 | Harvard Lectures on Pragmatism: Lecture V | CP 5.145

These three kinds of reasoning are Abduction, Induction, and Deduction. [–] Induction is the experimental testing of a theory. The justification of it is that, although the conclusion at any stage of the investigation may be more or less erroneous, yet the further application of the same method must correct the error. The only thing that induction accomplishes is to determine the value of a quantity. It sets out with a theory and it measures the degree of concordance of that theory with fact. It never can originate any idea whatever. No more can deduction. All the ideas of science come to it by the way of Abduction.

1903 | Harvard Lectures on Pragmatism: Lecture VI | CP 5.171

Abduction is the process of forming an explanatory hypothesis. It is the only logical operation which introduces any new idea; for induction does nothing but determine a value, and deduction merely evolves the necessary consequences of a pure hypothesis.

Deduction proves that something *must* be; Induction shows that something *actually is* operative; Abduction merely suggests that something *may be*.

Its only justification is that from its suggestion deduction can draw a prediction which can be tested by induction, and that, if we are ever to learn anything or to understand phenomena at all, it must be by that this is to be brought about.

1903 | Harvard Lectures on Pragmatism: Lecture VI | CP 5.168

However, it is now time for me to pass to the consideration of Inductive Reasoning. When I say that by inductive reasoning I mean a course of experimental investigation, I do not understand experiment in the narrow sense of an operation by which one varies the conditions of a phenomenon almost as one pleases. We often hear students of sciences, which are not in this narrow sense experimental, lamenting that in their departments they are debarred from this aid. No doubt there is much justice in this lament; and yet those persons are by no means debarred from pursuing the same logical method precisely, although not with the same freedom and facility. An experiment, says Stöckhardt, in his excellent *School of Chemistry*, is a question put to nature. Like any interrogatory, it is based on a supposition. If that supposition be correct, a certain sensible result is to be expected under certain circumstances which can be created, or at any rate are to be met with. The question is, Will this be the result? If Nature replies “No!” the experimenter has gained an important piece of knowledge. If Nature says “Yes,” the experimenter’s ideas remain just as they were, only somewhat more deeply engrained. If Nature says “Yes” to the first twenty questions, although they were so devised as to render that answer as surprising as possible, the experimenter will be confident that he is on the right track, since

2 to the 20th power exceeds a million.

1903 | Harvard Lectures on Pragmatism: Lecture VII | EP 2:234

I have argued in several of my early papers that there are but three essentially different modes of reasoning: Deduction, Induction, and Abduction. I may mention in particular papers in the *Proceedings of the American Academy of Arts and Sciences* for April and May 1867. I must say, however, that it would be very easy to misunderstand those arguments. I did not at first fully comprehend them myself. I cannot restate the matter tonight, although I am very desirous of doing so, for I could now put it in a much clearer light. I have already explained to you briefly what these three modes of inference, Deduction, Induction, and Abduction, are. I ought to say that when I described induction as the experimental testing of a hypothesis, I was not thinking of experimentation in the narrow sense in which it is confined to cases in which we ourselves deliberately create the peculiar conditions under which we desire to study a phenomenon. I mean to extend it to every case in which, having ascertained by deduction that a theory would lead us to anticipate under certain circumstances phenomena contrary to what we should expect if the theory were *not* true, we examine the cases of that sort to see how far those predictions are borne out.

1903 | Lowell Lectures on Some Topics of Logic Bearing on Questions Now Vexed. C. S. Peirce's Lowell Institute Lectures. 1903, Seventh Lecture. Introduction Vol. I | CP 7.110-120

Suppose we define Inductive reasoning as that reasoning whose conclusion is justified not by there being any necessity of its being true or approximately true but by its being the result of a method which if steadily persisted in must bring the reasoner to the truth of the matter or must cause his conclusion in its changes to converge to the truth as its limit. Adopting this definition, I find that there are three orders of induction of very different degrees of cogency although they are all three indispensable.

The first order of induction, which I will call *Rudimentary Induction*, or the Pooh-pooh argument, proceeds from the premiss that the reasoner has no evidence of the existence of any fact of a given description and concludes that there never was, is not, and never will be any such thing. The justification of this is that it goes by such light as we have, and that truth is bound eventually to come to light; and therefore if this mode of reasoning temporarily leads us away from the truth, yet steadily pursued, it will lead to the truth at last. [—]

The second order of induction consists in the argument from the fulfillment of predictions. [—] [—]

The third order of induction, which may be called Statistical Induction, differs entirely from the other two in that it assigns a definite value to a quantity. It draws a sample of a class, finds a numerical expression for a predesignate character of that sample and extends this evaluation, under proper qualification, to the entire class, by the aid of the doctrine of chances. The doctrine of chances is, in itself, purely deductive. It draws necessary conclusions only. The third order of induction takes advantage of the information thus deduced to render induction exact.

1903 | Lowell Lectures on Some Topics of Logic Bearing on Questions Now Vexed. Eighth Lecture, Abduction | CP 5.590

If we are to give the names of Deduction, Induction, and Abduction to the three grand classes of inference, then Deduction must include every attempt at mathematical demonstration, whether it relate to single occurrences or to “probabilities,” that is, to statistical ratios; Induction must mean the operation that induces an assent, with or without quantitative modification, to a proposition already put forward, this assent or modified assent being regarded as the provisional result of a method that must ultimately bring the truth to light; while Abduction must cover all the operations by which theories and conceptions are engendered.

1903 | Syllabus: Nomenclature and Division of Triadic Relations, as far as they are determined | EP 2:298; CP 2.269

An *Induction* is a method of forming Dicent Symbols concerning a definite question, of which method the Interpretant does not represent that from true premisses it will yield approximately true results in the majority of instances in the long run of experience, but does represent that if this method be persisted in, it will in the long run yield the truth, or an indefinite approximation to the truth, in regard to every question. An Induction is either a *Pooh-pooh Argument*, or an *Experimental Verification of a general Prediction*, or an *Argument from a Random Sample*. A Pooh-pooh Argument is a method which consists in denying that a general kind of event ever will occur on the ground that it never has occurred. Its justification is that if it be persistently applied on every occasion, it must ultimately be corrected in case it should be wrong, and thus will ultimately reach the true conclusion. A Verification of a general Prediction is a method which consists in finding or making the conditions of the prediction and in concluding that it will be verified about as often as it is experimentally found to be verified. Its justification is that if the Prediction does not tend in the long run to be verified in any approximately determinate proportion of cases, experiment must, in the long run, ascertain this; while if the Prediction will, in the long run, be verified in any determinate, or approximately determinate, proportion of cases, experiment must in the long run, approximately ascertain what that proportion is. An Argument from a Random Sample, is a method of ascertaining what proportion of the members of a finite class possess a predesignate, or virtually predesignate, quality, by selecting instances from that class according to a method which will, in the long run, present any instance as often as any other, and concluding that the ratio found for such a sample will hold in the long run. Its justification is evident.

1905 | Letters to Mario Calderoni | CP 8.209

... there are but three elementary kinds of reasoning. [—] The third way of reasoning is *induction*, or experimental research. Its procedure is this. Abduction having suggested a theory, we employ *de*duction to deduce from that ideal theory a promiscuous variety of consequences to the effect that if we perform certain acts, we shall find ourselves confronted with certain experiences. We then proceed to try these experiments, and if the predictions of the theory are verified, we have a proportionate confidence that the experiments that remain to be tried will confirm the theory. I say that these three are the only elementary modes of reasoning there are. I am convinced of it both *a priori* and *a posteriori*. The *a priori* reasoning is contained in my paper in the Proceedings of the American Academy of Arts and Sciences for April 9, 1867. I will not repeat it. But I will mention that it turns in part upon the

fact that induction is, as Aristotle says, the inference of the truth of the major premiss of a syllogism of which the minor premiss is made to be true and the conclusion is found to be true, while abduction is the inference of the truth of the minor premiss of a syllogism of which the major premiss is selected as known already to be true while the conclusion is found to be true. Abduction furnishes all our ideas concerning real things, beyond what are given in perception, but is mere conjecture, without probative force. Deduction is certain but relates only to ideal objects. Induction gives us the only approach to certainty concerning the real that we can have. In forty years diligent study of arguments, I have never found one which did not consist of those elements. The successes of modern science ought to convince us that induction is the only capable *imperator* of truth-seeking. Now pragmaticism is simply the doctrine that the inductive method is the only essential to the ascertainment of the intellectual purport of any symbol.

1906 [c.] | Suggestions for a Course of Entretiens leading up through Philosophy to the Questions of Spiritualism, Ghosts, and finally to that of Religion | MS [R] 876:4

The third kind of reasoning may fairly be called *Induction*, although it does not agree precisely with any one of the meanings attributed to this word. I apply it to any method of inquiry which is justified as a provisional result of a method which in the long run must lead to the truth. But such justification has several grades.

1906-7 | PAP [ed.] | NEM 4:319

Let us now consider non-necessary reasoning. This divides itself, according to the different ways in which it may be valid, into three classes: probable deduction; experimental reasoning, which I now call Induction; and processes of thought capable of producing no conclusion more definite than a conjecture, which I now call Abduction. [—] The general principle of the validity of Induction is correctly stated in the Johns Hopkins Essay, but is too narrowly defined. All the forms of reasoning there principally considered come under the class of Inductions, as I now define it. Much could now be added to the Essay. The validity of Induction consists in the fact that it proceeds according to a method which though it may give provisional results that are incorrect will yet, if steadily pursued, eventually correct any such error. The two propositions that all Induction possesses this kind of validity, and that no Induction possesses any other kind that is more than a further determination of this kind, are both susceptible of demonstration by necessary reasoning.

1907 | Pragmatism | MS [R] 318:30

As for induction (in my sense of this term) or the experimental method, it consists only in putting very definite questions to Nature, to which Dame Nature only replies, either by a curt “No,” or else by a dubious and provisional “Well.” No vestige, no scrap, of a new idea can possibly come from that source.

1908 | A Neglected Argument for the Reality of God | CP 6.475

… Observe that neither Deduction nor Induction contributes the smallest positive item to the final conclusion of the inquiry. They render the indefinite definite; Deduction Explicates; Induction evaluates: that is all.

1908 | A Neglected Argument for the Reality of God | CP 6.474

Concerning the question of the nature of the logical validity possessed by Deduction, Induction, and Retroduction, which is still an arena of controversy, I shall confine myself to stating the opinions which I am prepared to defend by positive proofs. [—] Induction is a kind of reasoning that may lead us into error; but that it follows a method which, sufficiently persisted in, will be Inductively Certain (the sort of certainty we have that a perfect coin, pitched up often enough, will *sometime* turn up heads) to diminish the error below any predesignate degree, is assured by man's power of perceiving Inductive Certainty. In all this I am inviting the reader to peep through the big end of the telescope; there is a wealth of pertinent detail that must here be passed over.

1908 | A Neglected Argument for the Reality of God | CP 6.472-473

The purpose of Deduction, that of collecting consequents of the hypothesis, having been sufficiently carried out, the inquiry enters upon its Third Stage, that of ascertaining how far those consequents accord with Experience, and of judging accordingly whether the hypothesis is sensibly correct, or requires some inessential modification, or must be entirely rejected. Its characteristic way of reasoning is Induction. This stage has three parts. For it must begin with Classification, which is an Inductive Non-argumentational kind of Argument, by which general Ideas are attached to objects of Experience; or rather by which the latter are subordinated to the former. Following this will come the testing-argumentations, the Probations; and the whole inquiry will be wound up with the Sentential part of the Third Stage, which, by Inductive reasonings, appraises the different Probations singly, then their combinations, then makes self-appraisal of these very appraisals themselves, and passes final judgment on the whole result.

The Probations, or direct Inductive Argumentations, are of two kinds. The first is that which Bacon ill described as "*inductio illa quæ procedit per enumerationem simplicem*." So at least he has been understood. For an enumeration of instances is not essential to the argument that, for example, there are no such beings as fairies, or no such events as miracles. The point is that there is no well-established instance of such a thing. I call this Crude Induction. It is the only Induction which concludes a logically Universal Proposition. It is the weakest of arguments, being liable to be demolished in a moment, as happened toward the end of the eighteenth century to the opinion of the scientific world that no stones fall from the sky. The other kind is Gradual Induction, which makes a new estimate of the proportion of truth in the hypothesis with every new instance; and given any degree of error there will *sometime* be an estimate (or would be, if the probation were persisted in) which will be absolutely the last to be infected with so much falsity. Gradual Induction is either Qualitative or Quantitative and the latter either depends on measurements, or on statistics, or on countings.

1908 [c.] | A Neglected Argument for the Reality of God (G) | CP 2.769

The true guarantee of the validity of induction is that it is a method of reaching conclusions which, if it be persisted in long enough, will assuredly correct any error concerning future experience into which it may temporarily lead us. This it will do not by virtue of any deductive necessity (since it never uses all the facts of experience, even of the past), but because it is manifestly adequate, with the aid of retroduction and of deductions from retroductive suggestions, to discovering any *regularity* there may be among experiences, while *utter irregularity is not surpassed in regularity by any other relation of parts to whole*, and is thus readily discovered by induction to exist where it does exist, and the amount of departure therefrom to be mathematically determinable from observation where it is imperfect.

1908 [c.] | A Neglected Argument for the Reality of God (G) | CP 2.756-759

It is well to distinguish three different varieties of induction. The first and weakest kind of inductive reasoning is that which goes on the presumption that future experience as to the matter in hand will not be utterly at variance with all past experience. *Example*: "No instance of a genuine power of clairvoyance has ever been established: So I presume there is no such thing." I promise to call such reasoning *crude induction*. [—]

From the weakest kind of induction let us pass at once to the strongest. This investigates the interrogative suggestion of retroduction, "What is the 'real probability' that an individual member of a certain experiential class, say the S's, will have a certain character, say that of being P?" This it does by first collecting, on scientific principles, a "fair sample" of the S's, taking due account, in doing so, of the intention of using its proportion of members that possess the predesignate character of being P. This sample will contain none of those S's on which the retroduction was founded. The induction then presumes that the value of the proportion, among the S's of the sample, of those that are P, probably approximates, within a certain limit of approximation, to the value of the real probability in question. I propose to term such reasoning *Quantitative Induction*. [—]

The remaining kind of induction, which I shall call *Qualitative Induction*, is of more general utility than either of the others, while it is intermediate between them, alike in respect to security and to the scientific value of its conclusions. In both these respects it is well separated from each of the other kinds. It consists of those inductions which are neither founded upon experience in one mass, as Crude Induction is, nor upon a collection of numerable instances of equal evidential values, but upon a stream of experience in which the relative evidential values of different parts of it have to be estimated according to our sense of the impressions they make upon us.

1909-12-25 | Letters to William James | EP 2:502

The third kind of warrant is that which justifies the use of a method of inference provided it be carried out to the end consistently. There are three kinds of inference of this kind. They are all inferences from random samples. The strongest is that which is a sample (that is, a collection) of units. In that case, the theory of errors is applicable. The second kind is where there are no definite multitudes but where, as the sample is enlarged, the inference becomes stronger and stronger. The third kind, which is the weakest of all forms of Induction, is where the only defence is that if the conclusion is false, its falsity will *sometime* be detected if the method of inference be persisted in long enough. [—] No inductive inference can be weaker than that and have any warrant at all.

1910 [c.] | Letters to Paul Carus | CP 8.236

…the essential character of induction is that it infers a *would-be* from actual singulars. These singulars must, in general, be finite in multitude and then […] the inductive conclusion can be (usually) but *indefinite*, and can never be *certain*…

1910 [c.] | On the Three Kinds of Reasoning [R] | MS [R] 755:9-11

…by Induction, or Inductive Reasoning, I mean all reasoning which infers that something is true of the whole of a given collection (no matter what kind of a collection it may be) because it has been found that the same thing is true of a part of that collection. [—] Pure inductive reasoning hardly ever, if indeed it ever does draw an altogether new and unanticipated conclusion. Its usual and proper business is to decide between inconsistent theories which we have been led to entertain by Retroductive Reasoning in order to account for different surprising phenomena. For we shall find that Induction is as far as Deduction from introducing into its conclusion any concept not already contained in its premisses. The true function of Induction is *critical*.

1911 | Letter to J. H. Kehler | NEM 3:178

… An Induction can hardly be sound or at least is to be suspected usually, unless it has been preceded by a Retroductive reasoning to the same general effect. Induction chiefly serves to render more certain ideas that have already been otherwise suggested. I use “Induction” in a wider sense than usual. It is usually regarded as a reasoning by which one passes from asserting something of a number of single things to asserting the same of the whole class to which those things belong. I give the definition a somewhat different turn, at least, and throw much light upon Induction by defining it as any reasoning from a *sample* to the whole sampled.

1911 | A Logical Criticism of the Articles of Religious Belief | MS [R] 856:3

By Induction, I mean a reasoning which provisionally concludes something to be true of every member of a collection, or, more frequently, of whatever there *may be* of a definite general kind, for no other reason than that firstly, the same thing has been found to be true of a part of that collection, or a finite collection of things of that kind, and secondly, that the manner in which this partial collection has come to be known to have the character which is concluded to belong to the whole, compels, or at least authorises, us to regard it, provisionally, approximately, and probably, as an image of that whole.

1913 | Letters to F. A. Woods | CP 8.385-387

I have always, since early in the sixties, recognized three different types of reasoning, viz: 1st, *Deduction* which depends on our confidence in our ability to analyze the meanings of the signs in or by which we think; 2nd, *Induction*, which depends upon our confidence that a run of one kind of

experience will not be changed or cease without some indication before it ceases; and 3rd, *Retroduction*, or Hypothetic Inference, which depends on our hope, sooner or later, to guess at the conditions under which a given kind of phenomenon will present itself.

Each of these three types occurs in different forms requiring special studies.

From the 1st type to the 3rd the security decreases greatly, while the uberty as greatly increases ... .

nd | Logic: Fragments [R] | MS [R] S64

There are three stages of inquiry, demanding as many different kinds of reasoning governed by different principles. They are,

1, *Retroduction*, forming an explanatory hypothesis[;]
2, *Deduction*, tracing out the consequences that would ensue upon the truth or falsity of that hypothesis; and
3, *Induction*, the experimental testing of the hypothesis by inquiring whether its consequences are born out by fact, or not.