# *Mathematics*

1895 [c.] | On Quantity, with special reference to Collectional and Mathematical Infinity | MS [R] 14:4

*Mathematics* is the study of the substance of hypotheses with a view to the tracing of necessary conclusions from them.

1895 [c.] | On Quantity, with special reference to Collectional and Mathematical Infinity | MS [R] 14:3

...the proper definition of mathematics is that it is the study of the substances of hypotheses, which it first frames and then traces to their consequences.

1895 [c.] | On Quantity, with special reference to Collectional and Mathematical Infinity | NEM 4:272-273

Each science (except mathematics) rests upon fundamental principles drawn from the truths discovered by the science immediately preceding it in the list, while borrowing data and suggestions from the discoveries of those which follow it.
1. *Mathematics*, which observes only the creations of the mathematician himself. It borrows suggestions from all other sciences, from philosophy Mathematical Logic, from psychics Mathematical Economics, from physics Mathematical Optics, Metrics, etc.
2. *Philosophy* ...

1895 [c.] | On Quantity, with special reference to Collectional and Mathematical Infinity | NEM 4:267-268

...the mathematician is not concerned with real truth, but only studies the substance of hypotheses. [—] ...the mathematician observes nothing but the diagrams he himself constructs; and no occult compulsion governs his hypotheses except one from the depths of mind itself.

Thus, the distinguishing characteristic of mathematics is that it is the scientific study of hypotheses which it first frames and then traces to their consequences. [—]

Mathematics is [...] the study of the substance of hypotheses, or mental creations, with a view to the drawing of necessary conclusions.

1895 [c.] | On the Logic of Quantity, and especially of Infinity | MS [R] 16:1

Mathematics may be defined as the study of the substance of exact hypotheses. It comprehends
1st, the framing of hypotheses, and
2nd, the deduction of their consequences.

1895 [c.] | Elements of Mathematics | NEM 2:10

… the mathematicians duty has three parts, namely,

1st, acting upon some suggestion, generally a practical one, he has to frame a supposition of an ideal state of things;

2nd, he has to study that ideal state of things, and find out what would be true in such a case;

3rd, he has to generalize upon that ideal state of things, and consider other ideal states of things differing in definite respects from the first.

This description of the mathematician's duty gives the best notion of what mathematics is: *it is the exact study of ideal states of things*.

1896 | The Regenerated Logic | CP 3.428

Mathematics is the most abstract of all the sciences. For it makes no external observations, nor asserts anything as a real fact. When the mathematician deals with facts, they become for him mere "hypotheses"; for with their truth he refuses to concern himself. The whole science of mathematics is a science of hypotheses; so that nothing could be more completely abstracted from concrete reality.

1896 | On the Logic of Quantity | MS [R] 13:1

The first prime division [of science] consists of Mathematics, which merely frames hypotheses and traces out their consequences, and alone of all the sciences does not concern itself with positive truth.

1896 [c.] | Lessons of the History of Science | CP 1.53

The most abstract of all the sciences is mathematics. That this is so, has been made manifest in our day; because all mathematicians now see clearly that mathematics is only busied about *purely hypothetical questions*. As for what the truth of existence may be the mathematician does not (qua mathematician) care a straw. It is true that early mathematicians could not clearly see that this was so. But for all their not seeing it, it was just as true of the mathematics of early days as of our own.

1897 [c.] | On Multitude | MS [R] 26:1

Mathematics is a study of exact hypotheses, in so far as consequences can be deduced from them. To limit mathematics to the deduction of those consequences would be to separate from it some of the greatest of the achievements of modern mathematicians, – achievements which nobody but mathematicians could have performed, – such as the formation of the idea of the system of imaginaries, and of the idea of Riemann surfaces. It must be allowed, therefore, that the formation of the hypotheses is a part of the business of mathematics.

1902 | Minute Logic: Chapter III. The Simplest Mathematics | CP 4.229

It was Benjamin Peirce, whose son I boast myself, that in 1870 first defined mathematics as “the science which draws necessary conclusions.” This was a hard saying at the time; but today, students of the philosophy of mathematics generally acknowledge its substantial correctness.

1902 | Minute Logic: Chapter II. Prelogical Notions. Section I. Classification of the Sciences (Logic II) | CP 1.239-240

Among the theoretical sciences, I distinguish three classes, all resting upon observation, but being observational in very different senses.

The first is mathematics, which does not undertake to ascertain any matter of fact whatever, but merely posits hypotheses, and traces out their consequences. It is observational, in so far as it makes constructions in the imagination according to abstract precepts, and then observes these imaginary objects, finding in them relations of parts not specified in the precept of construction. This is truly observation, yet certainly in a very peculiar sense; and no other kind of observation would at all answer the purpose of mathematics.

1903 | A Syllabus of Certain Topics of Logic | CP 1.183-187

Science of Discovery is either, I. Mathematics; II. Philosophy; or III. Idioscopy.

Mathematics studies what is and what is not logically possible, without making itself responsible for its actual existence. Philosophy is *positive science*, in the sense of discovering what really is true; but it limits itself to so much of truth as can be inferred from common experience. Idioscopy embraces all the special sciences, which are principally occupied with the accumulation of new facts.

1903 | Lowell Lectures. 1903. Lecture 3 | MS [R] 459:1-2

…if we conceive a science, not as a body of ascertained truth, but, as the living business which a group of investigators are engaged upon, which I think is the only sense which gives a natural classification of

sciences, then we must include under mathematics everything that is an indispensible part of the mathematician’s business; and therefore must include the *formulation* of his hypotheses as well as the tracing out of their consequences.

1903 | Lecture 5,. Vol. 2 | MS [R] 470:130-132

Mathematics, in general, is the science of the logical possibility & impossibility of hypotheses,

1903 | Lowell Lectures. 1903. Lecture 3. 1st draught | MS [R] 458:3-4

Mathematics is the science of hypotheses, – the science of what is supposable. Supposable does not mean directly imaginable, it means what makes sense.

1903 [c.] | On Dyadics: the Simplest Possible Mathematics | MS [R] 3:1

Mathematics will here be understood to be the science which sets up hypotheses with a view to doing what it proceeds to do, namely, to deduce their consequences, and to study the methods of doing so.

1904 | A Brief Intellectual Autobiography by Charles Sanders Peirce | Peirce, 1983, p. 70; MS [R] L107:18

Mathematics merely traces out the consequences of hypotheses without caring whether they correspond to anything real or not. It is purely deductive, and all necessary inference is mathematics, pure or applied. Its hypotheses are suggested by any of the other sciences, but its assumption of them is not a scientific act.

This quote has been taken from Kenneth Laine Ketner's 1983 reconstruction of Peirce's 'Autobiography'

1904 | Reason's Conscience: A Practical Treatise on the Theory of Discovery; Wherein logic is conceived as Semeiotic | NEM 4:191; HP 2:825

*Mathematics* is the study of what is or is not logically possible, without undertaking to ascertain what actually exists.

1907 | Second Talk to the Phil. Club [and] Second Talk. On Deduction | MS [R] 754

Mathematics is the practice of deduction.

1907-10-15 | The Prescott Book | MS [R] 277

Mathematics wholly relates to hypothetical states of things, perfectly exact, but of a kind that only affect a few very general characters.

nd | Notes on B. Peirce's Linear Associative Algebra | MS [R] 78:4

In short, logic is the theory of all reasoning, while mathematics is the practice of a particular kind of reasoning.

nd | A Suggested Classification of the Sciences | MS [R] 1339:6

I divide the sciences of discovery into, 1, *Mathematics*, which traces out the consequences of hypotheses without concerning itself with their truth, and as the business is carried on, also formulates these hypotheses to represent in some measure confused statements of supposed fact (or fancy); 2, *Philosophy*, which deals with positive truth, but only so far as it is discoverable from ordinary everyday experience; 3, *Idioscopy*, or Special Science, which business chiefly consists in observation.