Theorematic Reasoning

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1901 | On the Logic of Drawing History from Ancient Documents Especially from Testimonies (Logic of History) | EP 2:96

This appears to be in harmony with Kant's view of deduction, namely, that it merely explicates what is implicitly asserted in the premisses. This is what is called a half-truth. Deductions are of two kinds, which I call *corollarial* and *theorematic*. The corollarial are those reasonings by which all corollaries and the majority of what are called theorems are deduced; the theorematic are those by which the major theorems are deduced. If you take the thesis of a corollary, – i.e. the proposition to be proved, and carefully analyze its meaning, by substituting for each term its definition, you will find that its truth follows, in a straightforward manner, from previous propositions similarly analyzed. But when it comes to proving a major theorem, you will very often find you have need of a *lemma*, which is a demonstrable proposition about something outside the subject of inquiry; and even if a lemma does not have to be demonstrated, it is necessary to introduce the definition of something which the *thesis* of the theorem does not contemplate. In the most remarkable cases, this is some abstraction; that is to say, a subject whose existence *consists* in some fact about other things. Such, for example, are operations considered as in themselves subject to operation; *lines*, which are nothing but descriptions of the motion of a particle, considered as being themselves movable; *collections*; *numbers*; and the like. When the reform of mathematical reasoning now going on is complete, it will be seen that every such supposition ought to be supported by a proper postulate. At any rate Kant himself ought to admit, and would admit if he were alive today, that the conclusion of reasoning of this kind, although it is strictly deductive, does not flow from definitions alone, but that postulates are requisite for it.

1901 | On the Logic of drawing History from Ancient Documents especially from Testimonies (Logic of History) | NEM 4:1

It now appears that there are two kinds of deductive reasoning, which might, perhaps, be called *explicatory* and *ampliative*. However, the latter term might be misunderstood; for no mathematical reasoning is what would be commonly understood by *ampliative*, although much of it is not what is commonly understood as *explicative*. It is better to resort to new words to express new ideas. All readers of mathematics must have felt the great diffrence between *corollaries* and *major theorems*, although these words are not sharply distinguished. It is needless to say that the words come to us, not from Euclid, but from the editions of Euclid's elements. The great body of the propositions called corollaries (all but 27 in the whole 13 books) are due to commentators, and are of an obvious kind. Kant's characterization of all deductive reasoning is true of them: they are mere explications of what is implied in previous results. The same is true of a good many of Euclid's own theorems; probably the numerical majority of the whole 369 of them are of this character. But many are of a different nature. We may call the two kinds of deduction *corollarial* and *theorematic*.

1902 | Carnegie Institution Correspondence | NEM 4:49

My first real discovery about mathematical procedure was that there are two kinds of necessary reasoning, which I call the Corollarial and the Theorematic, because the corollaries affixed to the propositions of Euclid are usually arguments of one kind, while the more important theorems are of the other. The peculiarity of theorematic reasoning is that it considers something not implied at all in the conceptions so far gained, which neither the definition of the object of research nor anything yet known about could of themselves suggest, although they give room for it. Euclid, for example, will add lines to his diagram which are not at all required of suggested by any previous proposition, and which the conclusion that he reaches by this means says nothing about. I show that no considerable advance can be made in thought of any kind without theorematic reasoning. When we come to consider the heuretic part of mathematical procedure, the question how such suggestions are obtained will be the central point of the discussion.

[—] it is proper to divide all Theorematic reasoning into the Non-abstractional and the abstractional.

1902 | Carnegie Institution Correspondence | NEM 4:38

Deduction is divisible into sub-classes in various ways; of which the most important is into Corollarial and Theorematic. *Corollarial deduction* is where it is only necessary to imagine any case in which the premisses are true in order to perceive immediately that the conclusion holds in that case. All ordinary syllogisms and some deductions in the logic of relatives belong to this class. *Theorematic deduction* is deduction in which it is necessary to experiment in the imagination upon the image of the premiss in order from the result of such experiment to make corollarial deductions to the truth of the conclusion. The subdivisions of theorematic deduction are of very high theoretical importance.

1902 | Carnegie Institution Correspondence | NEM 4:42

Imagine, for example, an endless succession of objects. [—] Yet this proof will rest on some proposition which is simply self evident. But as long as one only has the idea of simple endless series, one may think forever, and not discover the theorem, until something suggests that *other* idea to the mind. What I call the *theorematic* reasoning in mathematics consists in so introducing a foreign idea, using it, and finally deducing a conclusion from which it is eliminated.

1902 | Minute Logic: Chapter III. The Simplest Mathematics | CP 4.233

How it can be that, although the reasoning is based upon the study of an individual schema, it is nevertheless necessary, that is, applicable, to all possible cases, is one of the questions we shall have to consider. Just now, I wish to point out that after the schema has been constructed according to the precept virtually contained in the thesis, the assertion of the theorem is not evidently true, even for the individual schema; nor will any amount of hard thinking of the philosophers' corollarial kind ever render it evident. Thinking in general terms is not enough. It is necessary that something should be DONE. In geometry, subsidiary lines are drawn. In algebra permissible transformations are made. Thereupon, the faculty of observation is called into play. Some relation between the parts of the schema is remarked. But would this relation subsist in every possible case? Mere corollarial reasoning will sometimes assure us of this. But, generally speaking, it may be necessary to draw distinct schemata to represent alternative possibilities. Theorematic reasoning invariably depends upon experimentation with individual schemata. We shall find that, in the last analysis, the same thing is true of the corollarial reasoning, too; even the Aristotelian "demonstration why." Only in this case, the very words serve as schemata. Accordingly, we may say that corollarial, or "philosophical" reasoning is reasoning with words; while theorematic, or mathematical reasoning proper, is reasoning with specially constructed schemata.

1903 | Syllabus: Nomenclature and Division of Triadic Relations, as far as they are determined | EP 2:297-298

A *Deduction* is an argument whose Interpretant represents that it belongs to a general class of possible arguments precisely analogous which are such that in the long run of experience the greater part of those whose premisses are true will have true conclusions. Deductions are either *Necessary* or *Probable*. Necessary Deductions are those which have nothing to do with any ratio of frequency, but profess (or their interpretants profess for them) that from true premisses they must invariably produce true conclusions. A Necessary Deduction is a method of producing Dicent Symbols by the study of a diagram. It is either *Corollarial* or *Theorematic*. A Corollarial Deduction is one which represents the conditions of the conclusion in a diagram and finds from the observation of this diagram, as it is, the truth of the conclusion. A Theorematic Deduction is one which, having represented the conditions of the conclusion in a diagram, performs an ingenious experiment upon the diagram, and by the observation of the diagram, so modified, ascertains the truth of the conclusion.

1907 | Pragmatism | MS [R] 318:55-6

In corollarial reasoning, the premisses act as stimulus to a suggestion according to general logical associations. But in theoric demonstration, it is necessary that associations should be introduced of which the premisses afford not the slightest hint. To this result two mental events must take place of natures as unlike each other as either is unlike an associative suggestion. Namely, in the first place, the ideas to be associated must be brought together in the mind, either by some accidental experience, or by the force of a natural or acquired instinct, or in consequence of a profound study of the forms of such associations. In any case, however, it will be a novel and original thought, cleverly shot upon the wing. In the second place an examination by means of experiments in the imagination must sufficiently show that the theoric association will involve no falsity.

1908 | A Neglected Argument for the Reality of God (O) | EP 2:441-442; CP 6.471

Deduction has two parts. [—] Explication is followed by Demonstration, or Deductive Argumentation. [—] It invariably requires something of the nature of a diagram; that is, an "Icon," or Sign that represents its Object in resembling it. It usually, too, needs "Indices," or Signs that represent their Objects by being actually connected with them. But it is mainly composed of "Symbols," or Signs that represent their Objects essentially because they will be so interpreted. Demonstration should be *Corollarial* when it can. An accurate definition of Corollarial Demonstration would require a long explanation; but it will suffice to say that it limits itself to considerations already introduced or else involved in the Explication of its conclusion; while *Theorematic* Demonstration resorts to a more complicated process of thought.

1909-12-25 | Letters to William James | EP 2:502

There are two kinds of Deduction; and it is truly significant that it should have been left for me to discover this. I first found, and subsequently *proved*, that every Deduction involves the observation of a Diagram (whether Optical, Tactical, or Acoustic) and having drawn the diagram (for I myself always work with Optical Diagrams) one finds the conclusion to be represented by it. Of course, a diagram is required to comprehend any assertion. My two genera of Deductions are first those in which any Diagram of a state of things in which the premisses are true represents the conclusion to be true and such reasoning I call *Corollarial* because all the corollaries that different editors have added to Euclid's *Elements* are of this nature. Second kind. To the Diagram of the truth of the Premisses something else has to be added, which is usually a mere May-be, and then the conclusion appears. I call this *Theorematic* reasoning because all the most important theorems are of this nature.