The evolution of Peirce's concept of induction

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Abstract:

The goal of the present work is to discuss the evolution Peirce’s concept of induction under the view of its correlation with realism. The process of induction in Peirce is grounded on the realism of the continua or the doctrine of synechism, in counterpart to the nominalistic, deterministic and necessitarian view of other authors.

Keywords: Induction

The main objective of this paper is to analyze the evolution of the concept of induction in the work of Peirce under the view of its correlation with realism, recalling that the validity of induction in Peirce results from the realistic hypothesis of his philosophy. The process of induction in Peirce is grounded on the realism of the continua or the doctrine of synechism, in counterpart to the nominalistic, deterministic and necessitarian view of other authors, mainly John Stuart Mill among them. My approach, therefore, escapes from the purely logical focus, turning to Peirce’s theory of reality as ground of the inductive argument.

According to Peirce, the inquiry starts from an uncomfortable state of doubt, which blocks the flow of usual actions, where it is not possible to choose among alternative courses of action. This doubt from which the inquiry departs is a real doubt, genuine and not just a methodological doubt, a make-believe.

Thus, the scientific inquiry constitutes an effort to put an end to the doubt and the truth would then be state of belief unapproachable by the doubt. The theory of inquiry can also be called the theory of scientific method. And for Peirce only the scientific method can lead us to the truth, in the long run, in a long way, which constitutes the dynamic process of inquiry. This process is subject to error and chance, but it can be self corrected in the long run and the inquiry has as a sole objective the agreement of opinions. The three stages of inquiry are: abduction, deduction and induction. This distinction grounds the theory of inquiry formalizing a cycle: abduction, deduction, induction, and a new abduction...

But going back to induction, this issue corresponds to logic and it constitutes one of the most important and difficult ones in terms of history of science because it deals with questions such as: which are the fundamental principles that we choose for the theories that have a predictive power or how can we make forecasts based on unobserved parts
for the whole universe? These questions refer to what we usually call induction.

The term induction was derived from the Latin translation *epagoge*, created by Aristotle, for whom the induction refers to all those cases of non-demonstrative arguments where the truth of the premises doesn’t require the truth of the conclusion. *Epagoge* means the establishing of the universal propositions, which can be expressed as in the form “all A are B”, due to the particular cases where this relation between A and B is valid. For Aristotle induction is the necessary starting point for the knowledge of what is universal (EN VI: 6).

But the first modern try to formulate a doctrine of the scientific method was due to Francis Bacon, in 1620, with the *Novum Organum*, where the title clearly suggests the ambition of Bacon to replace Aristotle’s *Organum* with a new logical instrument to bring progress to science. Bacon was the first philosopher who tried to formulate a theory of induction, appropriate for the natural sciences. He understood that the traditional logical was not an instrument of scientific discovery.

The great interest in the philosophy and methodology of induction was caused by the extraordinary success of the natural sciences. After Bacon, the classic authors who dealt with this subject started to realize that deduction did not have the power to turn explicit the logical consequences of generalizations. One of the main objectives of the natural sciences, when applying induction is to enable a rational inference from really observed matter into non observed and to foresee or infer the future. If the resource to intellectual intuition or self-evidence starts to be rejected as a source of factual knowledge, nothing else seems to be left but the trust in the empiric principle that all the knowledge referred to matters does, in fact, derive from the experience.

In this context, Hume states what has been called as “the great question of the induction”, which can be summed up as follows: what is supposed to be – by observation –structurally causal, will it so remain in the future, or how do we know that the future will be like the past? The question can be also put forward in the following way: are inductive conclusions valid? Science tries to find uniformities, but is there any reason to assume that the uniformities of the past will continue to happen in the future?

However, the experience conceived as sporadic, or as an indirect observation or as a systematic search for specific answers seems to provide knowledge only for particular truths and it was only at the end of the nineteenth century when some authors suggested solutions to demonstrate that the conclusion resulting from inductive arguments are valid.
As for the subject of induction, Mill (1806-1873) is one of the greatest speakers for Peirce. Mill was heir of a philosophy stemming from Locke, Berkeley and Hume and that was later developed by Hartley, Bentham and by James Mill, his own father. Mill’s writings on logic constitute an articulated and systematic formulation of the principles developed by the philosophy of the English sensationalist empirism and the utilitarianism.

According to Mill:

INDUCTION is the operation of the mind by which we infer that what we know to be true in a particular case or cases will be true in all cases which resemble the former in certain assignable respects. The mere summing up of details in a single proposition is not induction, but colligation; induction always involves inference from the known to the unknown, from facts observed to facts unobserved. (L, Introd. 2)

Mill maintained that the uniformity of the course of nature is the ultimate major premise of all inductions. Mill tended to speak almost interchangeably of the principle of uniformity and the law of causation or that every consequent is connected in this manner with some particular antecedent, or set of antecedents, characterizing a deterministic view of induction to which Peirce opposed his theory.

But before we approach the evolution into the proper sense of the Peircean concept of induction, it would be interesting to analyze the development of Peirce’s ideas with respect to the formation of inferences (deduction, induction and hypothesis), passing by the coincidence of the three arguments with three types of reasoning until we arrive at the breadth of the arguments in three stages of inquiry (abduction, deduction and induction).

In 1865 Peirce gave his first course at Harvard University (W1:165-167), called “The Logic of Science” and one year later, in 1866, he presented his first series of lectures “The Logic of Science or Induction and Hypothesis” at the Lowell Institute. In both the course and the lectures, his main objective was to show a non-psychological view of logic. Already in those days, Peirce introduced the hypothesis together with the induction, because he considered all mental processes as inferences, being the hypothesis an operation based upon data, contradicting a certainty valid for the time where there were only two types of arguments the deduction and the induction.

The introduction of the hypothesis, as one type of inference, constituted a revolutionary proposal of Peirce, who presented it as a type of inference subject to its own special rules, and therefore, opposite to the Cartesian ideas. It has to be noted, however, that at that time, Peirce still did not distinguish the deduction, induction and hypothesis as
three types of logical arguments different and irreducible between themselves (CP 8.337). Peirce was still under the strong influence of Kant, which led him to divide them in analytic (explicative) and synthetic (ampliative).

From 1865 to 67, Peirce worked on the text “On a New List of Categories” (CP 1.545-67, see Houser, 1992; Murphey, 1993), that would become a sort of a dorsal line to his logical doctrine, and which led him to adopt the three categories: quality, relation and representation. These categories were used to distinguish some inter-related triad constituting the structure of the logical system of Peirce: Three types of representations (or signs) -icon, index and symbol (CP 8.332); a triad of conceivable sciences -formal grammar, logic and formal rhetoric; a general division of symbols, common to all those three sciences (terms, propositions and arguments); three types of arguments distinguished by their three relations between premises and conclusion - deduction (symbol), induction (index) and hypothesis (resemblance) (See Santaella, 1992, pp. 69-70).

At the beginning of his works due to the influence of Kant, Peirce considered that every judgment consists in referring a predicate to a subject. The predicate is thought and the subject is only thought-of. The elements of predicate are experiences or representations of the experience and the subject is never experienced, only assumed (apud Murphey, 1993, p. 21).

For Peirce, therefore, all probable propositions have as a precedent a major premise and a minor premise (…) which are primal truths, these original premises do not have the nature of cognitions and yet, all the forms of inference, including the hypothesis could be reduced to the syllogism in Barbara. (CP 2.620)

Another issue that Peirce derives from Kant in his first works on the theory of knowledge is the doctrine that every cognition involves an inference, but if every cognition involves an inference, it is interesting to verify what type of inference would be there included. That differed from the traditional idea of inference used by most of the logicians, who considered it a cognition process whose standards are expressed by the arguments. But Peircean inference is an essential function of the mind, finally the life of thought, in all stages or situations, it is a question of exercise of certain habits of inference” (see Santaella, 1993, p.74).

From the studies of Aristotle and of Scotus, however, Peirce starts to discover “that there was something wrong with the formal logic of Kant” (CP 4.2, 1898), and even more when he reads “An Investigation of the Laws of Thought” by Boole. The Peircean
notion of inference evolved, and only when he was working “On a New List of Categories” Peirce found out that the three forms of inference couldn’t be reduced and they became three different and irreducible types of arguments (Serson, 1992, pp. 64-91).

Initially Peirce included the analogy as the fourth type of reasoning, but later he ended by recognizing that the analogy combines the characteristics of the induction and the retroduction. (CP 1.65, 7.98; Rescher, 1978). In 1867, in the first synthesis of “On a New List of Categories” (CP 2.461-618), presented at a meeting of the American Academy of Arts and Science, Peirce shows the correlation of the three forms of inference with three figures of the syllogism and proves that each figure involves an independent principle, the three figures are autonomous and its reduction is only obtained by limitations or denial (conversion and counterposition) and those are purely logical operations which have no real counterparts.

The inference form, according to Peirce, is an essential function of the mind (CP 1.372, 1.376, 2.711, 5.260, 5.267, 5.307, 5.327, 5.276, 6.416, 7.276, 7.354, 7.355, 7.357, 8.63, 8.144) and the thought in all levels presents a similar standard to the three types of arguments. For Peirce, an inference is a voluntary action, which culminates with the controlled adoption of a belief as a consequence of another knowledge. The first step in the inference usually consists in joining certain propositions which we believe to be true, the next step is the contemplation of an icon (observation) and a third one involves and compels the acceptance of a proposition related to it (judgment). An inference is a causal process, which creates or produces belief or its acceptance in mind (CP 2.442-44, 5.109).

Until the beginning of the seventies, the Peircean logic had still been grounded on the classical logic, specially in the subject-predicate theory or the proposition, but after the discovery of the logic of the relatives by the end of 1860, Peirce is led to introduce propositions that were not reduced to the form subject-predicate. After that, Peirce conceives three types of inference as distinguished and irreducible types of reasoning or arguments.

It is actually as far as 1878-79, when he enters Johns Hopkins, that logic becomes the method of the methods. Between 1890 and 1900, Peirce introduced new modifications, substituting hypothesis for abduction. The use of the word abduction is not original of Peirce. He is, however, the first author to employ it in a scientific context. Peirce translates apagoge of Aristotle into abduction, this means the acceptance or creation of the minor premise as a hypothetical solution for a syllogism whose major premise is
unknown and whose conclusion we find to be a fact (CP 7.249, CP 8.209; Anderson, 1987, p. 15).

For Peirce the division of every inference in abduction, deduction and induction can be almost presented as being the key of logic (CP 2.98). In 1898, his understanding of induction is modified and the term abduction is adopted as preferential, but the idea of abduction is quite complex and was only solved around 1901, when it becomes the process of formulating an explanatory hypothesis (CP 5.171).

Abduction constitutes the first stage of inquiry; the second stage deduction consists in deducing necessary consequences from the hypothesis (CP 2.755 or 6.469) and the third stage induction is the one where we establish to what extent these consequences are satisfied with the experience (MS 841; Misak, 1991, p. 94). While the abduction starts with a surprising fact generating a hypothesis, the induction starts with a hypothesis, leading it to be tested in experience. In this manner, the three types of inference become the three stages of inquiry, constituting the essence of the scientific method.

In the evolution of the Peircean thinking, it can be said that from 1903 onwards the questions referring to the classification of the sciences are established and it was around 1905 when the inter-relationship of the normative sciences is solved and in this context, the content itself and the objective of pragmatism is clarified. In parallel it can be added that this evolution is also due to the development of Peirce’s philosophy from nominalism towards extreme realism (CP 5.470, 8.208; Rescher, 1978; Santaella, 1993; Misak, 1991; Niklas, 1988; Burks, 1964)

The following discussion will consider as its ground the fives steps by which Fisch (1986, pp. 184-199) presents Peirce’s Progress from Nominalism toward Realism. These stages are: His initial nominalism, his first step toward realism, his second step toward realism, the pre-Monist period and the monist period.

1. His initial nominalism

The texts to be commented in this period are the following:

2. (1866) “Conscience and Language” (CP 7.580-81).
3. (1866) “The Conceptions of Quality, Relation and Representation applied to this Subject” (CP 2.422-26).
4. (1867) “On the Natural Classification of Arguments” (CP 2.451-2.516)
One of Peirce’s first text on induction is the “Harvard Conference II”, 1865, in which he presents his “general theory of induction”, directly derived from Aristotle doctrine. Aristotle says: “induction and the syllogism from induction is syllogysing one extreme as a predicate to the middle through the other extreme” (W1:175). The syllogysing consists in the synthesis in the inferred proposition of the two terms not united in either of the given proposition:

All carnivora are mammals
All mammals are vertebrates
\ All carnivora are vertebrates.

That induction is through simple enumeration, and Peirce objects to it saying that Aristotle supposes that a general term is equal to a sum of singulars. But this can be refuted, because singulars are not symbols, they are only signs and even they have extension, they have certainly no intention.

In Harvard Lecture X, 1865, Peirce shows that comprehension x extension = information (W1:272). Inference, in general obviously supposes symbolization and all symbolization is inference. He proves that induction, deduction and hypotheses are symbolyzable, but in relation to certainty deduction is the only demonstration; hypothesis is proverbially dangerous and the inductive argument became certain only by taking account of all that could possibly be known (W1: 283).

In “Conscience and Language”, 1868, Peirce explains that all modifications of consciousness are inferences and all inferences are valid. The valid inferences are:

- Intellectual inference with its three varieties Hypothesis, Induction and Deduction;
- Judgments of sensation, emotions, and instinctive motions which are hypotheses whose predicates are unanalyzed in comprehension; and Habits, which are Inductions whose subjects are unanalyzed in extension.

This division leads us to three elements of consciousness: 1st, Feelings or Elements of comprehension; 2nd, Efforts or Elements of extension; and 3rd, Notions or Elements of information, which is the union of extension and. (CP 7.58)

In 1866, Peirce presents eleven Lectures in Lowell, named “The Logic of Science or Induction and Hypothesis”, emphasizing the importance of logic. Lecture III shows that “deduction reasoning simply explicates our knowledge”, which is a faculty we are ready to attribute to intelect-to elaborate knowledge. But induction leads us to some new belief - as a dream does - and yet it is found generally to accord with the fact. Induction not merely explicates them but discovers new facts and yet those facts are
true. (W1:394)

Lowell Lecture V shows the common points between induction and hypothesis: both enlarge our knowledge, because their conclusions are not contained in their premises even implicitly (W1:425) and both explain facts (W1:426). The differences between induction and hypothesis are:

Induction stretches knowledge though to an infinite degree, a valid syllogistic conclusion and hypothesis is inferred from premises from which no syllogistic conclusion is valid; the induction is thus a little stronger argument than the hypothesis (W1:426).

Induction is the process by which we find general characters of classes and establish natural classifications (W1:426-427). Hypothesis alone affords us any knowledge of causes and forces, and enables us to see the why of things (W1:426)

Induction potentially increases the breadth of one term, and actually increases the depth of another, while hypothesis potentially increases the depth of one term, and actually increases the breadth of another. (CP 2.425).

Induction is a certain increase of breadth without a change of depth, by an increase of believed information. Abstraction is a decrease of depth without any change of breadth, by a decrease of conceived information. (CP 2.422)

The text “On the Natural Classification of Arguments” contains the first published discussion by Peirce on the synthetic inference, and its divisions the induction and the hypothesis. The hypothesis corresponds to the second figure of the syllogism and the induction corresponds to the third figure of syllogism.

We can say at that time that the three kinds of inference were differentiated by the categories of ampliation and differentiation. Induction and hypothesis enlarge our knowledge. Peirce also demonstrates the validity of the induction in syllogistic terms, departing from his definition as the inference of a major premise from a minor premise and a conclusion.

2. His first step toward realism

In the cognition articles Peirce develops a new theory for the mind and for the reality based on the results from “On a New List of Categories”. He gives the grounds for a really objective and empiric system of philosophy, in which the epistemology is founded in the representation of external facts, unifying in this way philosophy and science. Peirce’s central theme in this series is that all thinking is done by signs (CP 5.264), and
the cognition is a triadic relation between the subject and the object mediated by the sign. The theory of cognition from 1868-69 was completed in the theory of inquiry of 1877-78, joining the critics to the Descartes’ doctrine of intuition with a new development for grounding the inquiry based on the inferential concept of mind.

According to Fisch (1986, p.187), the first path toward realism is done in the cognition series. At that time the order for the inferences was deduction, induction and hypothesis according to their degree of certainty.

From the development of the theory of cognition, the theory of reality and the theory of inquiry, it results in a real conception which rejects the thing-in- itself, which is not restricted to the mental level nor to the individual level and although it derives from the external world, it has its truth guaranteed in the long run by the cognition process and by the consensus of the community. Peirce defines the real, introducing the idea of long run, the indefinite community and the independence from our representations: “In fact, it is the peculiar function of induction to produce universal and necessary propositions. Kant points out, indeed, that the universality and necessity of scientific inductions are but the analogues of philosophic universality and necessity”

Or

“I go to a fair and draw from the “grab-bag” twelve packages. Upon opening them, I find that every one contains a red ball. Here is a universal fact. It depends, then, on the condition of the experience. What is the condition of the experience? It is solely that the balls are the contents of packages drawn from that bag, that is, the only thing that determined the experience, was the drawing from the bag. I infer, then, according to the principle of Kant, that what is drawn from the bag will contain a red ball. This is induction. Apply induction not to any limited experience but to all human experience and you have the Kantian philosophy, so far as it is correctly developed.” (CP 5.223)

During this stage, Peirce still treats an inference as a syllogism. For him, a complete, simple, and valid argument was a syllogism that could be either apodictic or probable. An apodictic or deductive syllogism is one whose validity depends unconditionally upon the relation of the fact inferred to the facts posited in the premises. A syllogism whose validity should depend not merely upon its premises, but upon the existence of some other knowledge would be impossible, but a syllogism, whose validity depends partly upon the non-existence of some other knowledge is a probable syllogism and induction is defined as “the reasoning that proceeds as though all the objects which have certain characters were known.” And all valid reasoning is either deductive, inductive, or hypothetic. (CP 5.274)
So, induction may be defined as an argument which proceeds upon the assumption that all the members of a class or aggregate have all the characters which are common to all those members of this class concerning which it is known, whether they have these characters or not; or, in other words, which assumes that what is true of a whole collection is true of a number of instances taken from it at random. This might be called statistical argument. In the long run, it must generally afford pretty correct conclusions from true premises. The central characteristic and key to induction is that, by taking the conclusion so reached as major premise of a syllogism, and the proposition stating that such and such objects are taken from the class in question as the minor premise, the other premise of the induction will follow from them deductively. (CP 5.275)

The function of an induction is to substitute a series of many subjects for a single one that embraces them as well as an indefinite number of others. Thus it is a species of “reduction of the manifold to unity.” And just as induction may be regarded as the inference of the major premise of a syllogism, so hypothesis may be regarded as the inference of the minor premises, from the other two propositions. (CP 5.274-5)

In “Grounds for The Validity of The Laws of Logic: Further Consequences of the Four Incapacities” (CP 5.318-57), Peirce provides a justification four our general notions obtained through the inductive process, grounded on his theory of reality and also exposing his social theory of logic (CP 5.356).

He introduces some ideas on the induction, in relation to his cosmology and evolutionism. There seems to be a certain connection between our organs and the thing experienced, between our previous knowledge and what we learn that way, this connection is a faculty of man (CP 5.341). This is the only way that inductions are true and they are explained by the fact that the world and men have the same nature. Only this explains this trend that we achieve good inductions more often than bad ones (CP 5.353). When we believe in natural evolution we can think that men had a natural derivation. And it seems plausible to say that we inherited these faculties from a matrix, which is nature. Therefore, It is not worthless that our intelligence has some affinity with the natural forms if we believe in an evolutive derivation of our faculties and then there would be a very ancient real without the presence of human being (Ibri, 1997, p. 14).

Peirce asks how is synthetic reasoning possible at all? The answer to this question is the lock upon the door of philosophy. For Peirce “all probable inference, whether induction or hypothesis, is inference from the parts to the whole. It is essentially the same, therefore, as statistical inference.“ So the validity of this inference depends, in the long
run any one member of a class will occur as the subject of a premise of a possible induction as often as any other, and, therefore, the validity of induction depends simply upon the fact that the parts make up and constitute the whole. This in its turn depends simply upon there being such a state of things that any general terms are possible. And thus this part of the validity of induction depends merely on there being any reality and “from this it appears that we cannot say that the generality of inductions are true, but only that in the long run they approximate to the truth.”

On the point of view of the induction, when Peirce half opens “the door of philosophy”, he emphasizes some issues which will later become key to justify the induction. Those are the sign theory of knowledge or the theory of cognition, the theory of reality and the validity of synthetic inferences in the long run. The induction is characterized as the inference of a major premise from a minor premise and a conclusion; therefore, induction and hypothesis are synthetic inferences in the sense that the conclusion brings something which is not implied in the premise.

### 3. His second step toward realism

According to Fisch (1986, p. 188), the second step toward realism occurs in 1871, with the review of Fraser’s Edition “The works of George Berkeley” (CP 8.7-38). This review can be considered as a synthesis of the referring previous works on the theory of the cognition and theory of the reality, and also a development of the question of the realism-nominalism that may be resumed in the following question: Are universals real? There are six points to be emphasized in this text, which will be the ground for future developments.

1. Objects are divided into figments, dreams, etc., on the one hand, and realities on the other.
2. The real is that which is not whatever we happen to think of it, but is unaffected by what we may think of it.
3. The question, therefore, is whether man, horse, and other names of natural classes, correspond with anything which all men, or all horses, really have in common, independent of our thought, or whether these classes are constituted simply by a likeness in the way in which our minds are affected by individual objects which have in themselves no resemblance or relationship whatsoever.
4. This thing out of the mind, which directly influences sensation, and through sensation thought, because it is out of the mind, is independent of how we think it, and is, in short, the real.
5. All human thought and opinion contains an arbitrary, accidental element, dependent on the limitations in circumstances, power, and bent of the individual; an element of error, in short. But human opinion universally tends in the long run to a definite form, which is the truth.

6. There is, then, to every question a true answer, a final conclusion, to which the opinion of every man is constantly gravitating (…) The arbitrary will or other individual peculiarities of a sufficiently large number of minds may postpone the general agreement in that opinion indefinitely; but it cannot affect what the character of that opinion shall be when it is reached. This final opinion, then, is independent, not indeed of thought in general, but of all that is arbitrary and individual in thought; is quite independent of how you, or I, or any number of men think.” (CP 8.12)

4. The pre-Monist period (1872-1890)

In this section I will make reference to several articles such as:

2. (1878-79) “Illustrations of Logic of Science”: “The Fixation of Belief” - 1877 (CP 5.358-87), ·“How to Make our Ideas Clear“ - 1878 (CP 5.388-410), ·“The Doctrine of Chances “-1878 (CP 2.645-60), ·“The Probability of Induction “ - 1878 (CP 2.669-93), ·“The Order of Nature “ - 1878 (CP 6.395-427) and ·“Deduction, Induction, and Hypothesis “ - 1878 (CP 2.619-44), (these articles constitute part of the efforts of Peirce to solve the problem of the validity of the laws of logic.
3. (1882) “Introductory Lecture on the Study of Logic or Logic and Scientific Method” (W4:378:82, CP 7.59 -76)

According to Fisch (1986, p. 188), the pre-monist period corresponds to the major contributions to astronomy, geodesy, psychology, mathematics and mathematical logic. From the point of view of logic and mathematics, we have to emphasize Peirce’s works on the logic of relations and on truth-tables, indices and quantification, the resulting reformulation of his categories and his work on transfinite numbers. This period can be viewed as the combination of the idealistic theory of cognition with the theory of reality
aiming at to his mature realism with the development of his concepts of chance, continuity and evolution.

Peirce develops his theory of doubt-belief, from which the living doubt is the life of investigation and all inquiry presupposes a passage from a state of doubt to a state of belief. Because the only purpose of inquiry is the settlement of opinion, everyone who investigates, that is, who pursues an inquiry using the scientific method assumes that that process will, if carried far enough, lead him to a certain conclusion, he knows not what beforehand, but which no further investigation will change. No matter what his opinion may be at the outset, it is assumed that he will end in one predestinated belief.

Besides containing the first exposition of the pragmatism, the articles of the pre-Monist period still keep the continuity of the anti-Cartesian speech, but they present an advance in relation to the laws of the logic, mainly in that it relates to the hypothesis and induction and also they bring the explicit presence of the categories. In these essays the probability concept can be analyzed together with the concepts of validity of induction and the theory of reality, because probability is a fundamental tool for inductive logic.

According to Peirce we can classify all inference as follows:

\[
\text{Inference.} \\
\begin{array}{c|c}
\text{Deductive or Analytic.} & \text{Synthetic.} \\
\end{array}
\]

\[
\begin{array}{c|c}
\text{Induction.} & \text{Hypothesis.} \\
\end{array}
\]

It is interesting to note that Peirce made use of probability ideas in several parts of his philosophy, because he was impressed by the way frequencies tend to stabilize, so in an experiment, an observed relative frequency may be misleading at first but at the end settles down to a stable probability and induction is like that, because induction is self-correcting. The hypothesis of the reality bases the explanation of our ability to formulate valid inferences and the “truth consists in the existence of a real fact corresponding to the true proposition”, and indeed the validity of an inference consists in the truth of the hypothetic proposition that if the premises were true the conclusion will also be true.
By synthetic reasoning Peirce means those cases in which the facts summed up in the conclusion are not among those stated in the premises. These are the only inferences, which increase our real knowledge, however useful the others may be. But it is perhaps significant to note that all knowledge comes from synthetic inference. Though a synthetic inference cannot be by any means reduced to deduction, yet that the rule of induction will hold good in the long run may be deducted from the principle that reality is only the object of the final opinion to which sufficient investigation would lead. That belief gradually tends to fix itself under the influence of inquiry is, indeed, one of the facts with which logic sets out.” (CP 2.692-93)

So, Peirce defines induction as:

Induction is where we generalize from a number of cases of which something is true, and infer that the same thing is true of a whole class. Or, where we find a certain thing to be true of a certain proportion of cases and infer that it is true of the same proportion of the whole class.” (CP 2.624)

By induction, we conclude that facts, similar to observed facts, are true in cases not examined. Induction is reasoning from particulars to the general law, it classifies. (CP 2.636)

Induction is, plainly, a much stronger kind of inference than hypothesis (CP 2.642)

An induction is really the inference of a rule (Peirce, CP 2.629). (...) Now, the belief of a rule is a habit. That a habit is a rule active in us is evident. That every belief is of the nature of a habit, in so far as it is of a general character, has been shown in the earlier papers of this series. Induction, therefore, is the logical formula, which expresses the physiological process of formation of a habit. (CP 2.643)

In “A Theory of Probable Inference”, transposing the minor premise and conclusion, Peirce proves that induction and Hypothesis are apagogical inversions of statistical deductions where from they are valid as syllogism (CP 2.722-5). And he gives two rules for the synthetic inference: the first special rule for synthetic inference is that the sampling must be fair or the sample should be drawn at random and independently from the whole lot sampled and the second special rule is that of predesignation of characters (CP2.725-35) Peirce demonstrates that the induction is the inversion of the deduction statistics, of that its validity elapses.

We can say that most of the Pre-Monist phase is characterized by the so-called realism of one alone category (according to Fisch), that it is the realism of the thirdness. There is however a step of extreme importance that is the recognition of the necessity of the
indices for the adequate notation to the representation of the reasoning, because the real world cannot be distinguished from the world of the imagination for any description, from there the necessity of pronouns and indices.

5. The Monist period (1891-1914)

During the Monist period, Peirce publishes four article series:

1. The first “The Monist Metaphysical Series”, was published between 1891-93, including five articles.
2. The second monist series was in form of reviews on Schröder’s algebra and the logic of the relatives.
3. The third monist series includes the Pragmatism essays, published between 1905-1906.
4. The forth series is known as “Amazing Mazes”, and was published between 1908-1909.

It was in the Monist period that Peirce called himself an Aristotelian of scholastic wing (CP 5.77). His realism is now a “extreme realism” or a realism of three categories, including the reality of firstness because for Peirce, reality is not confined to actual existence, but must take account of possible qualities and potential effects, which has important consequences for his philosophy, among which we could emphasize his evolutionism, objective idealism and pragmatism. This period corresponds to the major changes in the concept of induction.

Peirce was the first author to suggest that the evolution is a similar process to the logic of the induction and similar to the creative process of the logic of the discovery. The peircean evolutionism is composed of three doctrines: tychism (absolute chance), synechism (continuous growth) e agapism (evolutionary love). These three doctrines can be related to abduction, deduction and induction so that tychism may be understood as a cosmic generalization of abduction, that is the logic of Discovery or the process of formulation of hypotheses and the synechism may be seen like a generalization of induction because induction is a form of training involving adaptation and generalization in the form of laws (beliefs and habits). But tychism and synechism are not sufficient to explain all the evolutionary process, so the agapism is the guide, without which the nature would evolve simply from one chaos to another. (Foster, 1997)

From the development and understanding of the normative sciences, Peirce’s pragmatism can be viewed as a step in the general procedure of the synechism and the pragmatism, therefore, strengthens the general character of realism, through the would-be’s because the reality of the general is the reality of the would-be’s. The would-
be’s are relevant to the final opinion because they refer to the ultimate end of inquiry, and Peirce characterizes them as something that could be realized. Further, the would-be’s and the generals are realities, which are crucial to Peirce’s opposition to nominalism. Pragmatism also establishes relations between the three types of reasoning, being a maxim of logical analysis (CP 6.490, 1908). But we cannot forget Peirce’s theory of probability, by which the concept of probability is closely related to the conditional “would-be”, a would-be has a property, quite analogous to any habit that a man might have” (CP 2.664-5). From 1905 onwards Peirce refines mathematically the concepts of long run and limit and the main characteristics of induction are the following:

Induction tends to correct itself. (CP 5.576 or RLT :167)

The justification of induction lies on the random sampling method and on the predesignation of the characters. (CP 2.269, 2.666-272)

Induction consists in starting from a theory, deducing from it predictions of phenomena, and observing those phenomena in order to see how nearly they agree with the theory. The justification for believing that an experiential theory which has been subjected to a number of experimental tests will be in the near future sustained about as well by further such tests as it has hitherto been, is that by steadily pursuing that method we must in the long run find out how the matter really stands.

We cannot forget its character experimental and provisional.

Induction has three types: crude, qualitative and quantitative

**Conclusion**

To conclude I would like to emphasize that Peirce’s ideas evolved from inferences (in which their order was deduction, induction and hypothesis according to their certainty degree), passing by the coincidence of the three arguments with three types of reasoning until we arrive at the breadth of the arguments in three stages of inquiry (abduction, deduction and induction).

According to Fisch, Peirce was above all a logician and his lifelong concern was the validity of synthetic inference, the logic of science, the logic of hypothesis and induction. His great lifelong ambition was to establish hypothesis and induction firmly and permanently along with deduction. Peirce believed that by adopting the scientific method, which means trusting abductive suggestions and submitting them to inductive testing, it was a good strategy to contribute to scientific progress. But in the long run
how must induction lead to the truth? Peirce’s reply is that induction is a method, which will always correct its error that may temporarily lead us into. And one of his last definitions for induction is the following:

The true guarantee of the validity of induction is that it is a method of reaching conclusions which, if it be persisted in long enough, will assuredly correct any error concerning future experience into which it may temporarily lead us. This it will do not by virtue of any deductive necessity (since it never uses all the facts of experience, even of the past), but because it is manifestly adequate, with the aid of retroduction and of deductions from retroductive suggestions, to discovering any regularity there may be among experiences... (CP 2.769).

References

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