Abstract:

Peirce introduced a conception of probabilities as “would-be’s” that are intensional, dispositional, directly related to the long run, and indirectly related to the single case. The most adequate conception takes them to be intensional, dispositional, directly related to the single case and indirectly related to the long run. When probabilities are properties of single cases, then finite “short runs” and infinite “long runs” are successively longer and longer sequences of single cases. In its general conception, if not its specific details, Peirce thus appears to have anticipated the resolution of one of the most difficult problems in the theory of science. This chapter elaborates Peirce’s contribution and explains the benefits of its single-case alternative in relation to crucial problems in quantum mechanics, evolutionary biology, and cognitive science, including connectionism and the philosophy of mind.

Keywords: Propensity, Probability, Frequency, Long Run, Short Run, Single Case, Quantum Mechanics, Evolutionary Biology, Cognitive Science

In his “Notes on the Doctrine of Chances” (CP 2.661) and related reflections, Charles S. Peirce advanced a conception of probabilities according to which a die and tossing device, for example, possesses “would-be’s” for its various possible outcomes, where these would-be’s are intensional, dispositional, directly related to the long run, and indirectly related to singular events. Among the most influential contemporary accounts—the frequency, the personal, and the propensity—the most promising, the propensity theory, provides an account according to which probabilities are intensional, dispositional, directly related to singular events, and indirectly related to the long run. Thus, in his general conception, if not its specific details, Peirce appears to have anticipated what seems to be the most adequate solution to one of the most difficult problems in the theory of science.

During his lifetime, Peirce shifted from the conception of probabilities as long-run frequencies to the conception of probabilities as long-run dispositions, that is, as tendencies to produce long-run frequencies. Section I below outlines Peirce’s conception of probabilities as long-run dispositions. Section 2 sketches the superiority of this view over its long-run frequency and personality probability alternatives. Section 3 explains the necessity to displace the conception of probabilities as long-run dispositions by one of probabilities as single-case dispositions. And section 4 suggests how this successor to Peirce’s account can contribute to the solution of contemporary scientific
problems. In passing, we shall consider how Peirce’s earlier views are related to his later views. Although a thinker’s later views do not always improve upon his earlier opinions, in this case Peirce’s later views turn out to be more adequate.

I. Peirce’s Conception

If the theory of probability had its origin in “games of chance” involving tosses of coins, throws of dice, draws of cards, and the like, then it is entirely appropriate that Peirce illustrated his conception by using this example:

I am, then, to define the meanings of the statement that the probability, that if a die be thrown from a dice box it will turn up a number divisible by three, is one-third. The statement means that the die has a certain “would-be”; and to say that a die has a “would-be” is to say that it has a property, quite analogous to any habit that a man might have. Only the “Would-be” of the die is presumably as much simpler and more definite than the man’s habit as the die’s homogeneous composition and cubical shape is simpler than the nature of the man’s nervous system and soul. (CP 2.664)

In this passage, Peirce characterizes probability as a dispositional property of a specific type of physical arrangement (such as a die and tossing device), explicitly invoking the subjunctive mood concerning “what would happen if.” Since this disposition is probabilistic, its effects are complex.

Now in order that the full effect of the die’s “would-be” may find expression, it is necessary that the die should undergo an endless series of throws from the dice box, the result of no throw having the slightest influence upon the result of any other throw... . It will be no objection to our considering the consequences of the supposition that the die is thrown an endless succession of times ... that such an endless series of events is impossible, for the reason that the impossibility is merely a physical ... impossibility. (CP 2.665-66)

Having already established that probabilities are supposed to be dispositional properties whose nature requires the subjunctive mood for its characterization, Peirce relates this intensional conception to its long-run displays.

The fact [is] that the probability of the die turning up a three or a six is not sure to produce any determination [of] the run of numbers thrown in any finite series of throws. It is only when the series is endless that we can be sure that it will have a particular character. Even when there is an endless series of throws, there is no syllogistic certainty, no “mathematical” certainty ... that the die will not turn up a six obstinately at every throw ... . It sanely would not, however, unless a miracle were performed; and moreover if such a miracle were worked, I should say (since it is my use of the term “probability” that we have supposed to be in question) that during this ... series the die took
The normal, nonmiraculous habit that a probability represents, Peirce explains, displays the outcome of interest with a limiting frequency which equals its generating probability when subjected to an endless sequence of trials. The relation involved here, moreover, is not logical but causal, insofar as these probabilities are displayed by corresponding limiting frequencies but are not defined by means of them. Indeed, as Peirce emphasizes, a deviation from this correspondence could occur, but this is merely a logical and not a physical possibility: “I say it might, in the sense that it would not violate the principle of contradiction if it did” (CP 2.667). Peirce thus endorses a conception of probabilities according to which these properties are dispositional, intensional, and directly related to their long-run displays.

II. Alternative Conceptions

The frequency conception of probability itself draws a strong connection between probabilities and their long-run displays, but it is precisely the definitional relationship that Peirce is prudent to deny. Indeed, while the theory of probability as an abstract domain of pure mathematics can be successfully developed by means of the frequency definition, it has a serious defect in application to physical sequences. For any case in which an endless series (of tosses, of throws, of draws, or of whatever) does not occur during the world’s history (including every finite sequence), there are no physical probabilities that satisfy the frequency conception because there are no “long-runs”. Although it may initially appear as though the advantage lies on the other side, with respect to its applicability to physical domains, the frequency definition offers no competition. (On the frequency view, see, for example, Salmon, 1966.)

Indeed, the magnitude of this difficulty for the frequency conception can be appreciated in relation to the problem of assigning appropriate values to the occurrence of singular events, such as the next toss of this coin, the next throw with that die, and the next draw from this deck. Any property whose presence or absence makes a difference to the limiting frequency with which corresponding outcomes occur presumably ought to be taken into account in assigning specific cases to proper reference sequences, since otherwise their assigned values would not be appropriate. Since every singular event has to have properties that distinguish it from every other event, it becomes increasingly difficult to satisfy the implied desiderata of the existence of infinite sequences of repetitions of such events under increasingly specific descriptions. (See Reichenbach, 1949; Salmon, 1966; Fetzer, 1981, Ch. 4.)
The personal conception, by contrast, falls prey to a different but related problem. According to this interpretation, probabilities are subjective properties, not of chance arrangements (like dice and tossing devices) but of persons and their beliefs. Thus, these probabilities are construed as reflecting the degrees of belief specific agents have in specific propositions, where a belief might acquire a probability of 1 as a function of its indubitability or a probability of 0 as a function of its incredibility. Most propositions would fall somewhere in between as neither personally necessary nor personally impossible. The problem with this approach, however, is that “degrees of belief” could exist as properties of a specific agent’s beliefs, even in the absence of probabilistic properties of any chance arrangements in the world. (On the personal view, see, for example, Skyrms, 1986.)

This situation is exemplified by “pull tab” lotteries, in which the purchase of a ticket permits the participant to remove the covering from a description of a possible prize. If pulling the tab discloses the description of a prize, the participant wins that prize, but otherwise not. Such “lotteries” are “games of chance” in the sense that the purchase of a ticket creates the possibility of a prize, and only a fixed number of prizes can be won. These “lotteries,” however, are not “games of chance” in the sense that they assume the existence of probabilistic properties or of indeterministic causation in the physical world. They are altogether compatible with causal determinism and are only “probabilistic” in relation to their participants’ personal convictions. (See, for example, Savage, 1954; de Finetti, 1964; Jeffrey, 1965.)

III. The Single Case

If Peirce’s conception confronts difficulties of its own, it does not suffer in comparison with these alternatives for the purpose of understanding the occurrence of events in the physical world. His account improves upon the frequency conception because its applicability does not presuppose the existence of infinite sequences of individual trials. It improves upon the personal conception because it applies to events in the world rather than to degrees of belief that we might have about them. In both of these respects, Peirce’s views appear to be theoretically superior to those reflected by the alternative conceptions. Nevertheless, because it is directly related to the long run, it may not be entirely satisfactory in the solution that it affords for the single case.

That Peirce was aware of this difficulty is not in doubt. The problem is vividly expressed in his reflections on the nature of probabilistic inference.

"An individual inference must be either true or false, and can show no effect "1" of probability; and, ..."
therefore, in reference to a single case considered in itself, probability can have no meaning. Yet if a man had to choose between drawing a card from a pack containing twenty-five red cards and a black one, or from a pack containing twenty-five black cards and a red one, and if the drawing of a red card were destined to transport him to eternal felicity, and that of a black one to everlasting woe, it would be folly to deny that he ought to prefer the pack containing the larger proportion of red cards, although, from the nature of the risk, it could not be repeated. (CP 2.652)

Thus, if probability can have no meaning relative to any single case considered in itself, it becomes very difficult to understand how probability is to serve its role as a “guide in life” with respect to the explanation and the prediction of immediate events. As Niiniluoto (1989) notes, Peirce sought to resolve this problem within the context of “A Theory of Probable Inference” by introducing an argument form called Simple Probable Deduction.

(1) The proportion \( p \) of \( M \)'s are \( P \)'s.
S is an \( M \).
It follows, with probability \( p \), that \( S \) is a \( P \). (CP 2.696)

Here Peirce relates relative frequencies of events with truth frequencies of conclusions, which he exemplifies by means of the following illustration:

(2) About two percent of persons wounded in the liver recover.
This man has been wounded in the liver.
Therefore, there are two chances out of a hundred that he will recover. (CP 2.694)

(For important discussions, see Niiniluoto 1981, 1982, and now especially 1989.)

The rationale Peirce supplies for deriving such an inference, moreover, has two aspects. On the one hand, he maintains that “to say …that a proposition has the probability \( p \) means that to infer it to be true would be to follow an argument such as would carry truth with it in the ratio of frequency \( p \)” (CP 2.697). On the other hand, he maintains that “it is requisite, not merely that \( S \) should be an \( M \), but also that it should be an instance drawn at random from among the \( M \)'s,” where “randomness” is intended to be understood as a function of our personal belief that the case under consideration should not be treated as a special, atypical case (CP 2.696).

It is not difficult to see that Peirce’s account here is not entirely satisfactory. Even if truth frequencies of conclusions correspond to relative frequencies of events over the long run, that does not establish their relevance for the occurrence of singular events. Moreover, our belief that \( S \) is a typical instance of \( M \) surely does not make it one, I no matter the degree of conviction with which that belief happens to be held. Insofar as
Peirce intended *Simple Probable Deduction* as a pattern of inference for both explanation and prediction, it cannot succeed unless it is utilized on the foundation of an objective conception of physical probability that can resolve the problem of the single case without subjective entanglements. When probabilities are properties of single cases, however, then finite “short runs” and infinite “long runs” simply turn out to be successively longer and longer sequences of single cases.

An adequate resolution of this difficulty thus requires a conceptual revision according to which probabilities are now to be understood as characterizing the strength of the causal tendency for a single trial with a physical arrangement to bring about one or another of its various possible outcomes. The propensity for obtaining the outcome of eternal felicity by drawing a card from a deck would equal 25/26 if that were the strength of the tendency for that outcome to be brought about on a single trial of that arrangement. These single-case propensities, in turn, tend to generate relative frequencies as expectable outcomes of repetitious trials of similar arrangements, an approach that affords a suitable foundation for improving on Peirce’s account. (See, for example, Popper, 1957, 1959; but especially Fetzer, 1971, 1981.)

**IV. Single-Case Propensities**

Probabilities as “propensities” are understood as intensional, dispositional, directly related to singular events, and indirectly related to the long run. In order to suggest the contemporary significance of this successor to Peirce’s conception, consider the illumination that it may shed upon problems from some of the most important areas of current research in physics, in biology, and in psychology. The first of these concerns the difficulties posed by the apparent paradoxes of quantum mechanics, which can be illustrated by the Schrodinger “cat paradox.” The second concerns the definition of fitness as it occurs within evolutionary biology, especially in relation to the notions of survival and reproduction. And the third concerns the nature of the architecture of the mind in relation to the connectionist conception of the brain.

Schrodinger’s cat paradox envisions an arrangement consisting of a live cat within an opaque chamber connected to an electrical device activated by a Geiger counter linked to a radioactive substance. If decay then occurs and is registered by the Geiger counter (with probability 1/2, let us say), then an impulse is activated that electrocutes the cat. From the perspective known as the Copenhagen interpretation, until the chamber is actually opened and it is discovered to be dead or alive, the cat is presumed to be neither dead nor alive but rather somewhere in between, which is represented by a
superposition of psi-functions for both of these events prior to this observation being made. The process of taking a look and discovering, say, that the cat is dead is thus supposed to effectuate “the collapse of the wave packet,” whereby in effect the result of being dead or of being alive is brought about.

From the perspective of the propensity conception, the Schrodinger cat paradox appears to be generated by the personal point of view. It is only possible for a quantum outcome to be assigned a personal probability value between 0 and 1 when that specific outcome, such as a cat’s being dead, remains unknown. As soon as an agent becomes aware that the cat is dead, the agent’s degree of belief that that is the case has to change to 1! This result, moreover, is not peculiar to quantum contexts but is commonplace with respect to games of chance in general. The propensity interpretation takes the mystery out of situations like these, therefore, by eliminating the need for a “collapse of the wave packet” as reflecting a confusion between changes in degrees of belief, on the one hand, and in strengths of causal propensities, on the other. (See especially Popper, 1967, 1982; and Fetzer, 1983.)

The notion of fitness as it occurs in evolutionary biology is an especially important explanatory conception. The principal benefit of the analysis of fitness as a propensity is that it identifies fitness with a tendency to survive and reproduce rather than with its actual attainment. It thus becomes possible for comparative theoretical determinations of fitness to differ from actual survival and reproduction, where “the more fit” might have fewer offspring than “the less fit” as a permissible probabilistic outcome “by chance.” As a propensity, of course, fitness is relative to whatever factors affect the prospects for an individual’s survival and reproduction, which will normally include not only its physical attributes but also each element of its environment that contributes to its chance of surviving and reproducing offspring.

Moreover, various measures of fitness can be constructed on the foundation that the propensity conception provides, some of which introduce reliance upon averages or “means” within heterogeneous populations or across divergent environments. The problems that arise with frequency dependent causation, for example, where the adaptability of a trait can vary with the number of members of a species that possess it, should be properly understood as manifestations of the context-dependence of propensity values generally, rather than as problematic special cases. And the discovery that “mean propensities” may turn out to be the appropriate measures of evolutionary prediction in specific situations enhances the prospects for biology to catch up with physics in its reliance upon sophisticated average values. (See Mills and Beatty, 1979;
Connectionism, perhaps the most exciting development within the domain of cognitive science, exhibits a similar potential for benefiting from the propensity approach. Connectionism models the brain as a neural network of numerous nodes that are capable of activation. These nodes can be connected to other nodes where, depending on their levels of activation, they may bring about increases or decreases in the activation of those other nodes. These patterns of activation, in turn, can function as signs for the larger systems of which they are otherwise meaningless elements by coming to stand for other things as “signs” for those systems. These might include features of their internal states or of their external environments as a consequence of the ways these things can function as signs for those systems.

Since the connections that are established between these various subsymbolic neural networks depend not only upon their levels of activation but also upon their predispositions to connect with other elements, it may prove exceptionally promising to entertain these nodes as endowed with propensities. From this point of view, the brain turns out to be a complex arrangement of neural propensities to establish connections under various conditions of system stimulation, where patterns of activation themselves can function as causal antecedents of human behavior. Indeed, a theory of the mind which appears ideally suited to complement such a conception of the brain—according to which minds are semiotic, or “sign using,” systems—can be developed on the basis of Peirce’s theory of signs. The problems that might benefit from conceptions stimulated by his work seem almost endless. (See Rumelhart et al., 1986; Smolensky, 1988; Fetzer, 1988b, 1990, 1996, 2002.)

The notion that Peirce advanced thus anticipated what appears to be the most adequate solution to a wide variety of difficult problems in the theory of science. The fertility of the propensity approach, moreover, promises to penetrate other perplexing conundra, including the deeper differences between determinism and indeterminism. The emergence of chaos theory as a branch of mathematical inquiry, for example, invites clarification on the basis of the consideration that the strength of causal tendencies depends on, and varies with, the presence or absence of every property whose presence or absence makes a difference to their outcomes, no matter how minute. It even appears as though chaotic phenomena and normal games of chance are deterministic rather than indeterministic, as true propensities require—an enlightening but nevertheless ironic fate for Peirce’s account to endure. (See Hacking, 1980; Crutchfield et al., 1986; Gleick, 1988; Fetzer, 1988a.)
References


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