"Things Unreasonably Compulsory": A Peircean Challenge to a Humean Theory of Perception, Particularly With Respect to Perceiving Necessary Truths[1]

"Coisas Injustificadamente Compulsórias": Um Desafio Peirciano à Teoria Humiana da Percepção, Particularmente com Respeito à Percepção de Verdades Necessárias

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Abstract: Much mainstream analytic epistemology is built around a sceptical treatment of modality which descends from Hume. The roots of this scepticism are argued to lie in Hume's (nominalist) theory of perception, which is excavated, studied and compared with the very different (realist) theory of perception developed by Peirce. It is argued that Peirce's theory not only enables a considerably more nuanced and effective epistemology, it also (unlike Hume's theory) does justice to what happens when we appreciate a proof in mathematics.

Keywords: Peirce. Hume. Necessity. Naturalism. Modal epistemology. Mathematical proof. Percept. Perceptual judgment. Percipuum.

Resumo: *Boa parte da epistemologia analítica tradicional é construída em torno de um tratamento cético em relação à modalidade que descende de Hume. As raízes desse ceticismo são apontadas na teoria (nominalista) da percepção de Hume, que é escavada, estudada e comparada com a muito diversa teoria (realista) da percepção de Peirce. Sustenta-se que a teoria peirciana não somente possibilita uma epistemologia consideravelmente mais sutil e eficaz, mas também (o contrário da teoria humiana) faz justiça ao que acontece quando apreciamos uma prova em matemática.*

Palavras-chave: *Peirce. Hume. Necessidade. Naturalismo. Epistemologia modal. Prova matemática. Percepto. Juízo perceptual. Percipuum.*

1 The sections of this paper devoted to Hume owe a great deal to previous joint work with Professor James Franklin, Department of Mathematics, University of New South Wales—although any errors or misunderstandings are of course my own responsibility. I am also indebted to discussions with Ahti-Veikko Pietarinen, Matthew Moore, Vinicius Romanini and Jeff Downard.

> "[…] the word occult, despite conjuring images of devil worship, actually means 'hidden' or 'obscured.' In times of religious oppression, knowledge that was counterdoctrinal had to be kept hidden or 'occult,' and because the church felt threatened by this, they redefined anything 'occult' as evil, and the prejudice survived." (Dan Brown, *The Lost Symbol*, p. 50)

> "It is self-evident that every truth of pure mathematics is self-evident if you regard it from a suitable point of view." (Charles Peirce, *Pragmatism as a Principle and Method of Right Thinking*, p. 128).

Introduction

Mainstream analytic epistemology is unfortunately wedded to an absurdly sceptical treatment of *modality*, according to which nothing in our experience may teach us about this vital dimension of truth. Following David Hume, attempts to challenge this assumption are met with an intimidating charge of *anti-naturalism*: an intellectual region in which few contemporary philosophers wish to tread.[2] Demands for 'truth-makers' for modal claims have been issued (BLACKBURN, 1986).[3] In order to meet these demands putative truth-makers have even been spread across other universes allegedly entirely spatiotemporally disconnected from ours, and thus inaccessible to our experience (LEWIS, 1986).

This paper will critique this Humean legacy, arguing that it springs from uncritical acceptance of a certain kind of empiricism. The end of the paper will point towards an alternative modal epistemology drawn from the thought of Charles Peirce, which holds that we do learn modal truths through experience, although it requires that the concept of experience be broadened beyond the senses to include ideal objects experimented on using diagrams. The paper's key example will be taken from mathematics, which is a useful place to observe the limitations of the Humean legacy in modal epistemology since, as Peirce observed, following his esteemed father, mathematics is "[…] the science that draws necessary conclusions" (PEIRCE, *Collected Papers*, henceforth CP, 4.229). The resources Peirce's philosophy offers us will include a considerably more nuanced theory of *perception* than is found in Hume.

The overall structure of the paper is as follows. First (§1) we will examine a mathematical example, then (§2) explicate Hume's theory of perception, and (§3) its consequences for modal epistemology with respect to both matters of fact and relations between ideas. Following that, we will (§4) explore Peirce's theory of perception, (§5) argue that it does a better job of accommodating what actually happens when we understand such proofs, and (§6) conclude with some reflections on occult powers.

2 For a paradigmatic example, see JACKSON and PRICE, 1997.

3 See also HALE, 2002's inquiry into "the source of necessity".

1. The Phenomenology of Necessary Reasoning

Consider this simple diagrammatic proof of a proposition in arithmetic:

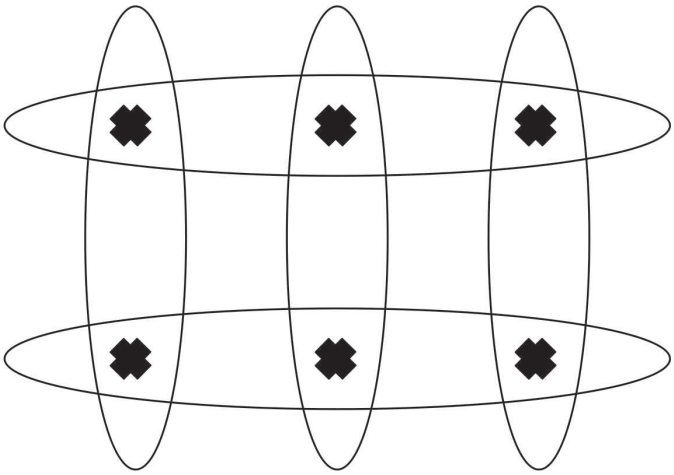

Figure 1. Why $2 \times 3 = 3 \times 2$

Grasping this proof turns on perceiving something, but what, exactly? For it seems that we are grasping not just that 2×3 *is* 3×2, but that 2×3 *must be* 3×2. It is clear that to try to instantiate another option, such as $2 \times 3 = 3 \times 3$, would be futile. It therefore *appears* that, at least in mathematics, we may perceive states of affairs that are necessarily true or, as it is sometimes put nowadays (making the phenomenon seem truly remarkable), true in all possible worlds.

We could prove the same mathematical proposition in a more stepwise, symbolic, 'inferential' manner, but this diagram seems to give us everything we need to directly perceive its necessary truth. To Peirce, with his profound appreciation of iconic signs, this is no accident since he believed all necessary reasoning was *diagrammatic*: concerning "[…] a diagram of our own creation, the conditions of whose being we know all about" (MOORE, 2010, p. 19). Thus Peirce would claim that more stepwise proofs are diagrammatic also—the diagrams merely less perspicuous.[4]

But how does it happen that we might perceive something and on that basis come to know a necessary truth? The very claim seems to violate important insights of empiricism, such as that "experience just shows us what is right in front of us"—namely actual concrete objects. At this point we might recall Hume's apparently devastating objection to the perception of causal necessity: that "[…] the understanding never observes any real connexion among objects, and […] even the union of cause and effect, when strictly examin'd, resolves itself into a customary association of ideas" (*Treatise*, 1, IV, vi, pp. 259-60).

4 At one time Peirce put this point by distinguishing between mathematical diagrams that are merely 'imputational' (e.g. algebraic proofs) and fully 'inherential' (e.g. graphical proofs like *Fig. 1*) (MOORE, 2010, p. 43).

In order to properly examine these matters, we first need to outline Hume's theory of perception, and the epistemology he twines around it.

2. Hume's Theory of Perception: "All ideas, which are different, are separable"

2.1. Ideas and Impressions

Hume offers an essentially mechanistic account of perception, built around a postulated direct causal contact between the mind and objects both 'internal' and 'external'. This contact somehow generates *impressions* and *ideas*. The difference between these two consists merely in "the degrees of force and liveliness, with which they strike upon the mind, and make their way into our thought or consciousness" (*Treatise*, 1, I, i, p. 1)[5]. Ideas are fainter, less "lively", *copies* taken by the mind of impressions. One might imagine this relation between impressions and ideas to be something like that of a stamp and its imprint (*Fig. 2*). This is obviously a metaphor, but one which arguably captures two important features of perception as Hume understands it. Firstly, the process is *direct* – it has no intermediary, for instance in other, rational, faculties of the mind. Ironically, this creates the famous 'veil of ideas', or Humean phenomenalism, since on this model one cannot 'think behind' one's impressions of the world and form other ideas about it, because all ideas are copies of impressions.[6] Nevertheless in the impression-forming process the mind is envisaged to directly confront the world, and surely this (in some form) is what perception *must* consist in?

Figure 2

Secondly, the process is *determinate*. Hume's impressions and ideas are particulars**,** and as such, possessed of a determinate set of features which it is assumed are copied from impressions to ideas precisely and wholesale. Hume argues for this claim phenomenologically by mentally comparing one of his impressions and its corresponding idea and arguing that no features have been lost: "When I shut my eyes and think of my chamber, the ideas I form are exact representations of the impressions I felt; nor is there any circumstance of the one, which is not to be found in the other" (*Treatise*, 1, I, i, p. 3).

One might legitimately query the introspective research methodology here. (If any features *had* been lost—how would Hume know?) It's also worth noting that Hume acknowledges that the point only applies to simple ideas, since his idea of the City of New Jerusalem is composed of a range of impressions that he has never perceived together. However my present aim is not to critique but merely illustrate Hume's philosophical account of perception. Another aspect of this understanding of ideas as direct, determinate 'stampings' on the mind is that they are also *temporal*

5 See also HUME, *Enquiry*, p. 18.

6 The recent sceptical realist interpretation of Hume by for instance John P. Wright and Galen Strawson arguably does not remove this veil of perception so much as install a 'know not what' behind it.

particulars in the sense that they occur at a particular time–point and there is no temporal duration within the idea itself. (Within the Humean perspective this very notion sounds bizarre and unmotivated. Nevertheless it will be returned to later).

A significant problem for any philosophy of perception is how to reconcile two aspects of the mind's encounter with the world in which it finds itself which seem rather different and opposed. On the one hand, my perceptions are suffused with immediately felt experience (for instance, the juicy, sweet 'cherryness' of a cherry I am biting into) which it seems that in some important sense 'no-one can take away from me'. Thus the nature of our sensory feels appears to enjoy some degree of infallibility ("Even if that cherry was a total hallucination, I can't be wrong about how it tasted to me."). On the other hand, much of the point of perception seems to be to enable us to endorse new propositions about the world that are truth-apt ("This cherry is delicious! But is it *really* a cherry, or rather a small plum?"). In this regard our perceptions seem perfectly fallible.

This is all rather confusing. We might dub this issue *The Experience-Truth Gap* in perception. In order to address it, in philosophy of perception one traditionally encounters talk of 'seemings', 'sense-data' and other like entities, which are identified as further objects of perception over and above the real-world objects allegedly being perceived.[7] What is said about the intermediary objects is then treated as bearing the full weight of perception's apparent infallibility, while what is said about the real-world objects is treated as bearing the full weight of perception's apparent fallibility.

However, postulating these intermediary objects arguably doesn't solve the problem at hand. If the role of representing sensory feels is given over entirely to the intermediary objects (which is what has tended to happen), delicate issues emerge concerning which of the qualities of those objects are 'primary' (had by both intermediary and real-world objects) and which merely secondary (had by the intermediary objects alone), ultimately leading to scepticism about whether there are any primary qualities at all, or any that can be known—a scepticism that has its logical conclusion in Kant's entirely unknowable *noumena*. Meanwhile, if the role of logically assessability is given over entirely to statements about the putative real world objects, we seem to be deprived of sufficient contact with them in order to assess them properly. (Of course these issues have been more than well-rehearsed in the literature).

British Empiricism seeks to paper over the breach to some degree by designing its key concept of an *idea* to play the dual role of both representing sensory feels and being logically assessable. It is sometimes remarked, particularly in the Lockean tradition, that ideas in their role as representing sensory feels are viewed as *caused by* the world, whilst they are logically assessable by virtue of *resembling* the world (Understanding ideas as copies of impressions is meant to ensure this second role of ideas as resembling the world). It has been probed to what degree these two roles of being caused by and resembling the world are compatible, or guaranteed to deliver the same results. We might also ask whether resemblance is able to 'do all the world-representing work' that might be required in our perceptual contact with our surroundings—more on this later.

7 For classic texts, see RUSSELL, 1912; AYER, 1958. For a particularly nuanced account, see SELLARS, 1982. For a probing critique of the framework, see MCDOWELL, 1994.

Understanding how Hume thinks about perception is important for understanding his epistemology in particular, since he claims that in an important sense, all functions of the mind reduce to it: "To hate, to love, to think, to feel, to see; all this is nothing but to perceive" (*Treatise* 1, I, ii, p. 67)[8]. Impressions may enter from 'outside the mind' (impressions of *sensation*) or be generated 'inside the mind' (impressions of *reflexion*), but the latter must consist in some combination of impressions that have previously entered by the senses, which are the building blocks of all thought.[9] It is by this kind of mental *passivity* that Hume imagines that he ensures empiricism.

Crucially, Hume holds that ideas and impressions are all essentially *distinct*. By this he means wholly separable in the imagination, if not in reality. Distinctness so understood is a key concept for Hume which we will examine further in the next section.

2.2. Hume's Naturalistic Rejection of Abstract Ideas

Hume's empiricist commitment to all ideas deriving ultimately from sensation means that he must hold that there are no *abstract ideas*. An example would be an idea of a triangle which is neither isosceles or scalene, and whose sides have no specific length (*Treatise*, 1, III, i). An abstract idea is thus an idea with at least some determinables which have not been rendered determinate (in the idea). Hume credits Berkeley with "one of the greatest and most valuable discoveries that has been made of late years in the republic of letters" in denying that such ideas exist, and claiming that "[…] all general ideas are nothing but particular ones, annexed to a certain term, which gives them a more extensive signification" (*Treatise*, 1, I, vii, p. 17). He claims, "whatever objects are separable are also distinguishable […]" (*Treatise*, 1, I, vii, p. 24), and by separable he means separable in thought. Moreover, he explains at length that the distinguishability he is speaking of means more than a scholastic "distinction of reason", which he disdains as "[…] so much talked of, and […] so little understood, in the schools […]". Rather, it means literally imagining (a perception-like experience of) objects which instantiate one idea and not the other.

For instance, when we distinguish shape from colour in an object such as a white cube, it is not that we use reason to distinguish its whiteness and its cubehood as abstract ideas. Rather, we imagine a cube of some other determinate colour (e.g. black) and a white thing with some other determinate shape (e.g. globular). Without such a literal, quasi-perceptual forcing apart of ideas, Hume claims, we cannot distinguish them, and thinking that we can is a cause of much confusion and wasted time in philosophy. Let us call this claim Hume's *Separate Imaginability Criterion of Distinctness*. This criterion basically consists in the denial that, when distinguishing ideas, one might abstract without separating.

After mocking the very notion of an abstract idea as a means for philosophers to "refuse to submit to the decisions of clear ideas", Hume argues against it as follows:

8 See also HUME, *Enquiry*, p. 152.

9 In *Treatise* 1,I, ii, p. 6-7, Hume mentions the formation of "secondary ideas, which are images of the primary", but assures the reader that these too derive ultimately from impressions.

> But to destroy this artifice, we need but reflect on that principle so oft insisted on, that all our ideas are copied from our impressions. For from thence we may immediately conclude, that since all impressions are clear and precise, the ideas, which are copied from them, must be of the same nature […]. An idea is by its very nature weaker and fainter than an impression; but being in every other respect the same, cannot imply any very great mystery. (*Treatise*, 1, I, iii, p. 72-3).

The basic structure of this argument seems to be that if ideas are perceived only by the intellect (or soul), then they must be mysterious, and if mysterious, they must be explanatorily inefficacious. This seems a rather weak argument. If an abstract idea is not fully reducible to impressions derived from experience, does it follow that it must be perceived *solely* by the intellect? This seems like arguing that because mothers cannot create babies on their own, fathers must.[10] Why is he arguing so lazily here? The answer is that the doctrine of abstract ideas is an artefact of scholastic realism, considered in Hume's day a degenerating research programme that had been miserably discredited, and thus an easy target for parody.

One major reason for such discrediting, in Hume's view, was the medieval Aristotelian philosophers' extravagant postulation of *final causes*—behaviour governed by ends or purposes (e.g. objects fall because they in some sense 'seek' to get closer to the earth). Such thinking, he stated, constituted a further lamentable failure to restrict our ideas purely to what may be derived from impressions:

> […] among all the instances, wherein the Peripatetics have shown they were guided by every trivial propensity of the imagination, no one is more remarkable than their sympathies, antipathies, and horrors of a vacuum. There is a very remarkable inclination in human nature, to bestow on external objects the same emotions, which it observes in itself; and to find everywhere those ideas, which are most present to it. This inclination, 'tis true, is suppress'd by a little reflection, and only takes place in children, poets, and the ancient philosophers […] (*Treatise*, 1, IV, iii, p. 224).

2.3. Hume's Naturalistic Rejection of Occult Qualities

Hume's denial of the mysterious ends and purposes thought to govern final causation is strongly connected with his denigration of so-called *occult qualities*. These are supposed hidden objects or powers in nature which when considered philosophically are shown to be defined by their unverifiability, and therefore to merit epistemic contempt. A good example, to which Hume delivers a thorough drubbing, is the Aristotelian idea of substance (or 'prime matter'):

> […] these philosophers carry their fictions still farther in their sentiments concerning occult qualities, and both suppose a substance supporting, which they do not understand, and an accident supported, of which

10 In the surrounding context Hume also seems to be equivocating on the meaning of 'mystery' – between on the one hand, profoundity or depth and on the other hand, incoherence.

> they have as imperfect an idea. The whole system, therefore, is entirely incomprehensible […] (*Treatise*, 1, IV, iii, p. 222).[11]

Not only is the concept difficult to understand, Hume charges that the way it is used encourages epistemic laziness, since scholastic philosophers: […] need only say, that any phenomenon that puzzles them, arises from an occult quality, and there is an end of all dispute and enquiry upon the matter" (*Treatise*, 1, IV, iii, p. 224).

This attack on the occult has been widely emulated by modern philosophers. And surely Hume is correct that to adopt a properly scientific attitude is to pledge to base all knowledge on observation, and since occult qualities are by definition unobservable, they must be, as Hume describes them, incomprehensible fictions? Who wants to be gullible to the kinds of mysterious powers that are peddled by the unscrupulous today (just as in Hume pointed out in his piece "Of Miracles"). Surely if we don't take Hume's views on board we will end up believing in something embarrassing like telepathy? This issue will be discussed further below.

3. Hume on Perceiving Necessity: "No necessary connections between distinct existences"

We turn now to Hume's sceptical treatment of the idea that we might perceive necessary truth. It has often been thought well-summarised in the maxim: "There are no necessary connections between distinct existences".[12] As so stated, the maxim sounds more metaphysical than epistemological, and indeed it is today widely taken for granted by metaphysicians as both a piece of obvious common-sense and an axiom of their discipline. Louis DeRosset provides a nice overview of this:

> It is difficult to overstate the influence of Humean scepticism about necessity on latter-day philosophers. One symptom of this influence is the centrality in contemporary philosophical debates of the Humean claim:
>
> **(Humean claim)** There are no necessary connections between distinct existences.
>
> Philosophers deploy this sweeping claim in the service of a wide variety of philosophical projects. It is advertised as an appropriate starting point for theorizing about what is necessary or possible. David Lewis (2001, p. 611) has even suggested that it is *the* starting point: it provides us with our best handle on what is possible (DEROSSET, 2009, p. 153).[13]

Let us review the maxim in action in Hume's discussion of two key areas: causation and mathematics. It will be useful to examine these together since they fall on either side of Hume's strict division of knowledge into *relations between ideas* which are determined *a priori,* and *matters of fact* which can only be learned through experience: a distinction sometimes referred to as *Hume's Fork*.

11 For a recent spirited defence of the scholastic idea of substance, composed with a fidelity to the Thomist worldview that is really worthy of note, see ODERBERG, 2009.

12 Hume never stated exactly these words, but came close enough, e.g. "No connexions between distinct existences are ever discoverable among human understanding." (*Treatise*, Appendix).

13 For another very useful recent exposition and critique of the claim, see WILSON, 2010.

3.1. Causation

Hume's first—and to his mind most important—application of the maxim is to explode philosophers' naïve ideas about *causal necessity*. Consider the famous scene where one billiard ball strikes another. One supposes that one can see the first ball *make* the second ball move. But strictly speaking, Hume argues, all we see is that the first ball moves towards the second, *and then* touches it briefly, *and then* the second ball moves away:

> I consider, in what objects necessity is commonly suppos'd to lie; and finding that it is always ascrib'd to causes and effects, I turn my eye to two objects suppos'd to be plac'd in that relation […]. I immediately perceive, that they are contiguous in time and place, and that the object we call cause precedes the other we call effect. In no one instance can I go any farther, nor is it possible for me to discover any third relation betwixt these objects. (*Treatise*, 1, II, xiv, p. 155)

Lacking any possible third relation between the two events in his experience, he concludes that causal necessity cannot be perceived.

In a way, the crucial move here is the "two objects". Hume treats the first ball touching the second and the second ball moving away (call these events **C** and **E** respectively) as 'distinct existences'. To justify the claimed lack of necessity between them he gives a *phenomenological argument*—we can readily imagine **C** without **E**:

> When I see, for instance, a Billiard-ball moving in a straight line towards another […] may I not conceive, that a hundred different events might as well follow from that cause? May not both these balls remain at absolute rest? May not the first ball return in a straight line, or leap off from the second in any line or direction? All these suppositions are consistent and conceivable. (*Enquiry*, IV, 1, 25, p. 13)

He then claims that only *constant conjunction* between events like **C** and events like **E** teaches that an instance of the former produces an instance of the latter in particular cases such as our billiard balls. But this 'production' is strictly speaking not any kind of necessary connection between any particular **C**-like and **E**-like event-pair considered in isolation. We merely *impute* production when experience presents us with a large enough collection of **C**-like events followed only by **E**-like events, so that "from the constant conjunction the objects acquire an union in the imagination", and "[w]hen the impression of one becomes present to us, we immediately form an idea of its usual attendant […]" (*Treatise*, 1, III, vi, p. 93). And nothing more may be said on the matter of perceiving causal necessity.

In philosophical discussions of causation Hume has very much won the day.[14] Theorists are by no means unanimous in their positive account, dividing between a '*Humean Regularity theory*' (e.g. DAVIDSON, 1980; KIM, 1973), *counterfactual theories* (LEWIS, 1973), and accounts in terms of *agency* (MENZIES

14 It is interesting at this point to contrast ethics, where Hume's denial of the reality of deontic necessity has not won the day nearly so much, Kantianism still being a strong contender.

and PRICE, 1993). But none of these positions claim that causal necessity is directly perceivable in the world. Certain philosophers have recently begun attacking so-called causal Humeanism in metaphysics, seeking to re-establish full-blooded causal necessity in the existence of essential properties, e.g. (ELLIS, 2001; LOWE, 2012). But there is scant discussion of the related epistemology—how we might come to know what essences things have, over and above what observable properties they have.

3.2. Mathematics

We noted that mathematics falls on the other side (than causation) of Hume's Fork. Hume's arguments against gaining knowledge of necessity in the realm of "matters of fact", which we have just reviewed, mean that he must relegate it (knowledge of necessity) to the other, purely ideal, side of his great divide, and think of it as discoverable by the mere operation of thought. Thus, although Kant himself was never so crude, the overall tradition of philosophy since Kant has been to assimilate Hume's 'relations of ideas' to analytic truths (so that a 20th century philosopher such as A. J. Ayer could spend a large part of his career endeavouring to discredit the synthetic *a priori*). The Fork has been problematic for mathematics insofar as it seems to entail that mathematics qua *a priori* science is entirely disconnected from real-world matters of fact. As Einstein put the received opinion: "[…] as far as the propositions of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality" (EINSTEIN, 1954, p. 233).

The early Hume even appears to deprecate the *a priori* science's accuracy. Here is what he says about geometry:

> [G]eometry […] never attains a perfect precision and exactness. Its first principles are still drawn from the general appearance of the objects; and that appearance can never afford us any security, when we examine the prodigious minuteness of which nature is susceptible. Our ideas seem to give a perfect assurance, that no two right lines can have a common segment; but if we consider these ideas, we shall find, that they always suppose a sensible inclination of the two lines, and that where the angle they form is extremely small, we have no standard of a right line so precise, as to assure us of the truth of this proposition (*Treatise*, 1, I, iii, pp. 70-71).

He does judge number theory to be more reliable because its objects are (in his view) countable and discrete (*Treatise*, 1, I, iii, p. 71). But he dismisses the concept of *infinity*, and thus any mathematical results based on it, because our ideas of quantity cannot be divided indefinitely:

> […] we really must […] regard all the mathematical arguments for infinite divisibility as utterly sophistical. For 'tis evident, that as no idea of quantity is infinitely divisible, there cannot be imagin'd a more glaring absurdity, than to endeavour to prove, that quantity itself admits of such a division (*Treatise*, 1, II, v, p. 52).

To be blunt, this epistemology produces a miserably impoverished view of mathematics, even in as it existed in Hume's day.[15] To give just one example: Hume's philosophical career postdates the giant achievements of calculus in showing how precise answers might be determined for infinitely long processes of calculation. One of its two key founders, Isaac Newton, was even meant to be Hume's great inspiration as a naturalistic philosopher. Today such a restricted view of mathematical certainty is insupportable, and it's worth noting that contemporary philosophy of mathematics rarely mentions Hume.

So how *can* we do justice to mathematical truth as a part of reality, but also answer the profound Humean epistemological challenge to explain how we obtain knowledge of necessary truth when all that our experience of the real world appears to present us with is actual concrete objects? In order to solve this epistemological problem, merely positing that "essential properties exist" (as do the modern-day essentialists alluded to above) is arguably not enough. Peirce's theory of perception, it will be claimed, has the answer.

4. Peirce's Theory of Perception: "[...] [N]othing at all [...] is absolutely confrontitional"

I will now discuss the detailed theory of perception Peirce developed around 1902-3, in the still largely unpublished *Minute Logic*, and a piece which the *Collected Papers* entitles "Telepathy and Perception", and which is most intriguing in the light of some themes discussed in the earlier part of this paper. This theory of perception differs greatly from Hume's. By contrast to the British Empiricists' use of *ideas* to model both immediate experience and truth-apt propositions derived from it, Peirce suggests that we need separate, though interlocking, accounts of these two things. The first becomes his account of the *percept*, the second his account of the *perceptual judgment*.

4.1. The Percept

The percept comprises a *felt quality* and the *vividness* with which it is presented. Neither of these is what nowadays would be called "cognitive". Peirce notes that one might call the percept an "image", except that an image is often taken to represent something other than itself, and the percept does not do that (CP 7.619). Nevertheless it has *insistency*: it makes a real impact on my consciousness.[16]

Peirce explicates the insistency of the percept along three dimensions. The first is that the percept contributes something positive to my thinking. If for instance I have a percept of a cat, I do not just perceive some abstract state of affairs such as the absence of any dog in that spatiotemporal region. I perceive

15 FRANKLIN, 1994 notes that Pascal cited "the Chevalier de Méré's belief in atomic space as proof of his total incompetence in mathematics" (p. 85), and: "in omitting his treatment of space and time almost entirely from the *Enquiry*, Hume seems to admit tacitly that it was not a success with its intended audience" (p. 86). See also FLEW, 1976; FOGELIN, 1985.

16 Peirce's percept bears some relation to a medieval concept of the *phantasm* (see for e.g. Aquinas, *Summa Theologiae*, Bk I, q. 85, art. 1). I am grateful to James Franklin for pointing out this reference to me.

something that possesses qualities of its own, such as colours, shapes, sounds. Secondly, the percept compels my thinking insofar as I cannot pretend that it is not present in my consciousness. Thirdly, the percept is not reasonable. By this Peirce does not mean that the percept is *ir-* so much as *a-*rational. He states, "[…] it does not address the reason, nor *appeal* to anything for support" (CP 7.622). For one thing, the percept does not have sufficient structure to be rationally evaluable, or for itself to be a rational evaluation. It does not have any parts, or more strictly, "It has parts, in the sense that in thought it can be separated, but it does not represent itself to have parts" (CP 7.625). Attentive Peirce scholars will recognise that in this explication he is making use of his three fundamental philosophical categories. He is affirming that Firstness and Secondness—and denying that Thirdness—pertain to the percept.

The percept in its directness bears some similarity to a Humean impression, but it cannot be a Humean idea insofar as it cannot be used to make truth-claims, nor is it the subject of belief or disbelief (CP 7.626). Peirce writes that it:

> […] does not stand for anything. It obtrudes itself upon my gaze; but not as a deputy for anything else, not 'as' anything. It simply knocks at the portal of my soul and stands there in the doorway (CP 7.619).

If the percept is really so mute, we might ask what is the point of positing it, epistemologically? Here we might look to the two roles that (it was noted) ideas have been said to play in British Empiricism: resembling and being caused by the world. We might ask whether the role of Peirce's percept is to *resemble* the world in the positive qualities that we have noted that it possesses. This is not the case, however. In Locke the claim that our ideas resemble objects entirely external to those ideas was a hypothesis empty of the very empirical consequences so beloved by empiricists—an insight decisively seized upon by Berkeley—and Peirce does not make it.

Perhaps, then, the percept might play a *causal* role in philosophy of mind, and related epistemology—perhaps analogous to the causal role Hume gives to impressions in producing ideas? There is some truth to this, insofar as Peirce claims that percepts are related to perceptual judgments by "forceful connections". Only, this forcefulness should not be understood as *the cause of a copy*, where that copy is a particular. Rather, it is *a trigger for* (general) *habits*. This claim will be explained further below.

4.2. The Perceptual Judgement

The perceptual judgement cannot be a copy of the percept, as they are too unlike one another. Peirce describes them as "[…] as unlike […] as the printed letters in a book, where a Madonna of Murillo is described, are unlike the picture itself" (CP 5.54. 1903 Harvard lectures, p. 160). Why is this? First of all the percept has an *integration* which cannot be possessed by the perceptual judgement, which *qua* judgement requires subject and predicate. Peirce offers as an example his perceiving a yellow chair (*Fig. 3*)[17]:

Figure 3

17 The reader is invited to imagine the diagram printed in colour.

> The judgement, 'This chair appears yellow', separates the color from the chair, making the one predicate and the other subject. The percept, on the other hand, presents the chair in its entirety and makes no analysis whatever (CP 7.631).

The percept also has a *definiteness* which conflicts with the general predication which a judgment must contain. Peirce analyses this definiteness into two dimensions. The first is that it is *individual*: the percept pertains to some particular chair and no other. The second is that the percept is *perfectly explicit*: all of its determinables are determinate (CP 7.625). Thus the yellowness of the chair-percept will be some perfectly specific colour, such as a dark lemon yellow, whereas our predicate 'yellow', due to its wide usage, must perforce be more general. It's worth noting that the generality of the yellow predicate is a kind of *specifically sensory* generality, which Peirce refers to a number of times using the metaphor of a 'composite photograph', a technology popular in his time which involved exposing the same negative to different objects in order to achieve a kind of 'visual average'[18]:

> Let us consider, first, the predicate, 'yellow' in the judgment that 'this chair appears yellow.' This predicate is not the sensation involved in the percept, because it is general. It does not even refer particularly to this percept but to a sort of composite photograph of all the yellows that have been seen (CP 7. 634).

A further dimension of the perfect explicitness of the percept is that whereas the perceptual judgment with its chosen colour-predicate makes no comment on other 'chair-determinables', such as shape, these will be discernible in the percept too.

Since the perceptual judgement is composed of subject and general predicate, thereby expressing a truth-apt proposition, its interpretation opens out to the community of inquiry. As Forster puts it, "While the content of a percept is inherent in it apart from everything else, the content of a sign is not" (FORSTER, 2011, p. 114). Rather, the perceptual judgment takes its (logical) place in "[…] an endless series of judgments, each member of which is logically related to prior members" (*Ibid*., p. 120). Thus in our example above, inquirers may develop the meaning of *yellow* and *chair* in unanticipated ways: for example, by determining the wavelength of light which typically produces yellow experiences in humans, or by inventing a chair which lacks some feature previously thought essential, such as legs.

Despite its pathways into public discourse, however, the perceptual judgement *compels assent* as much as the percept. It is equally insistent. As much as if I open my eyes in front of a yellow chair I cannot avoid having certain sensory experiences, neither can I avoid judging "This is a yellow chair", if I have the appropriate concepts. But how is it possible that the perceptual judgment produce such compulsive belief?

18 For a nice clear explication of this idea, and tracing of it through Peirce's philosophy, see HOOKWAY, 2002. One aspect Hookway does not highlight is that the composite photograph might be understood as capturing a distinctively iconic kind of generality which corresponds to Peirce's category of Firstness, as opposed to a more conceptual understanding of generality as 'extension of an idea' that might correspond to Peirce's category of Thirdness.

Doesn't this endow it with a form of *de facto* infallibility? We have just noted that the perceptual judgment opens out logically into the community of inquiry, for which it is well known that Peirce makes thorough fallibilism the guiding principle. Surely it cannot be both fallible and infallible at the same time?

This is an important objection. A superficial initial answer might point out a temporal dimension to the belief-forming process, and note that the perceptual judgment's apparent infallibility holds *at the time*, but it might be corrected subsequently in the light of further perceptions. ("For an instant I saw a yellow chair in the corner. But when I blinked and looked again I only saw floorboards. Therefore I infer that what previously appeared to me as a perception of a yellow chair was in fact a hallucination, and I choose to ignore it.") But shortly we will see that in Peirce's philosophy the temporal mediation of *what we perceive* applies on a yet profounder level.

Finally, despite the fundamental differences between the perceptual judgement and the percept that have been noted, the former nevertheless "professes to represent" the latter. In this representing function it embodies the Thirdness that is missing from the percept (CP 7.360). But one might wonder: how on earth is it possible for the perceptual judgment to represent the percept, if they are so different? We have just made clear that the perceptual judgment is not a copy of the percept. Peirce adds that neither does it represent the percept *logically*, since this would require that the percept serve as some kind of premise from which the perceptual judgment is inferred. As the percept is not itself in propositional form, it cannot serve as a premise for the perceptual judgment; nor can it be described in such a way that it could so serve, without recapitulating the perceptual judgment and begging the question (CP 7.628). The issue of the true relationship between percept and perceptual judgment will be resolved in the next section.

4.3. The Relationship between Percept and Perceptual Judgement: The Percipuum

Now it might seem that Peirce has so convincingly separated the Firstness / Secondness of the percept from the Thirdness of the perceptual judgement that one might wonder: how are we to bridge the two? In particular, how are we to bridge the *uncontrollable* in perception to the *controllable* in thought? Don't we now have a great mystery at the heart of perception? Isn't Peirce deeply entrenching the Experience-Truth Gap, rather than giving us any theoretical means to resolve it?

The answer is that the British empiricists (and their downstream followers) are too unimaginative in assuming that the only possible relation between percept and perceptual judgment (or in Humean terms: impression and idea) is that the latter *copies* the former. But *how* can the idea convey the same information as the impression, other than by copying it? Otherwise surely we would be engaged in some mere blind, causal transaction between our minds and the world? Peirce claims "[…] there is no relation between the predicate of the perceptual judgment and the sensational element of the percept, except forceful connections" (CP 7.634). The previously unanticipated third possibility for the relation between perceptual judgment and percept is that the former is an *index* of the latter—a "true symptom, just as a weather-cock indicates the direction of the wind or a thermometer the temperature" (CP 7.628).

How does this work? The human mind is organised such that each percept produces "direct and uncontrollable interpretations" (CP 7.648) which lead the mind to form various perceptual judgments. These interpretations are sometimes referred to by Peirce via a third term: the *percipuum*. Insofar as the percipuum consists in an interpretative welding of percept to perceptual judgment, it may be understood to manifest full-blooded Thirdness. So we may ask: with the percipuum have we finally bridged perception into the controllable in thought? Alas, no—Peirce notes that the percipuum is equally insistent!

> The percipuum […] is what forces itself upon your acknowledgment, without any why or wherefore, so that if anybody asks you why you should regard it as appearing so and so, all you can say is, 'I can't help it. That is how I see it.' (CP 7.643).

However although this interpretative process cannot be willed, it can be (indeed must be) trained and perfected by cultivating appropriate mental *habits*.[19] For example, parents spend considerable time training children to correctly apply predicates that are useful in their daily lives ("food", "bath", "red", "one", "two", "three"). Over time, those children learn how to produce appropriate judgments about the objects around them with the help of whatever perceptual experiences they notice reliably correlate with those judgments. But the exact nature of the experiences themselves—in Wittgenstein's famous phrase—may be 'divided through' as irrelevant. This process is broadly known as 'education'.

4.4. The Experience-Truth Gap Mediated

The answer to the Experience-Truth Gap in our philosophical understanding of perception is not to split the object of perception in two—postulating one object that is unreal but is the one that is actually perceived, and a second object that is real but 'lies behind' the first and is only inferred (the manifest problems of which for British Empiricism soon emerged). Rather than two objects, the answer is *time*. The percipuum is not a temporal particular. It occurs across a time-span which has at its 'back end' a memory of the immediate past (which Peirce calls the *ponecipuum*) and at its 'front end' an expectation of the immediate future (which he calls the *antecipuum*).

The ponecipuum is a kind of sediment of past perceptions, interpreted such as to trigger us to view current percepts in the categories required for us to make perceptual judgments. In the case of the yellow chair this will involve a synthesis of previous perceptions that have been judged to 'involve chairs'. At the heart of the ponecipuum lies a pure sensory *ponecept*, which in the case of our example will consist in some kind of generalization of all our past 'chair-like' and 'yellow-like' experiences, although once again it is strictly unable to be put into words (again the specifically iconic metaphor of a composite photographis appropriate).

The antecipuum is our present experience interpreted in terms of its most immediate predictive implications (e.g. "This object is good to sit on, and unlikely

19 In appreciating this point I have benefitted greatly from extended philosophical conversations with Joshua Black.

to change colour at random"). Under pragmatism, such hypothetical conditionals constitute the meaning of concepts and propositions. The antecipuum also rests on kernel of pure experience—an *antecept*. Rosenthal describes the antecept as an "element of vague, not fully conscious anticipation of future experience" (ROSENTHAL, 2001, p. 3), and notes that it (and indeed all these rather alarmingly proliferating entities in Peirce's theory of perception) is not a literal or consciously experienced stage in perception but "the abstraction of a 'stopping point'" in its logical analysis. In a more epistemological register, they are "[…] not the building blocks of perception but a verification level brought about by a change of focus when a problem arises." (*Ibid*., p. 4).

In order to explicate this last claim, we may now examine the deeper analysis of the fallibility of perceptual judgments that was promised in the last section. It is presented by Rosenthal in an acute analysis of this difficult passage by Peirce:

> Now let us take up the perceptual judgment "This wafer looks red." It takes some time to write this sentence, to utter it, or even to think it. It must refer to the state of the percept at the time that it, the judgment, began to be made. But the judgment does not exist until it is completely made. It thus only refers to a memory of the past; and all memory is possibly fallible and subject to criticism and control. The judgment, then, can only mean that so far as the character of the percept can ever be ascertained, it will be ascertained that the wafer looked red (CP 5.544, 1903).

Rosenthal interprets this as saying that the perceptual judgment is indubitable not in the sense that doubts about it can be answered with certain knowledge, but in the "pragmatic" sense that doubts about it cannot coherently be formulated:

> […] to doubt it is to put into question something for which there is no tool for getting "behind" it to compare it with anything more fundamental. For us it must itself be the final court of appeal. The apprehension of an appearance is not certainly true as opposed to possibly false. It is "certain" in the sense that neither truth nor falsity is applicable to it…for what the percipuum is determined only in its recognition and can be determined in no other way. It becomes a "repetition" of previous contents only by being assimilated to those contents in the perceptual judgment (ROSENTHAL, 2001, p. 4).

To explore this further, let us return to the case of the disappearing yellow chair percept. Our initial analysis of this scenario held that we have two distinct percepts: the first percept 'yellow-chair-like' and the second percept judged to represent only floorboards. On the basis of such a mismatch, so close together in time, I infer that the first percept is a hallucination and so I both remember and disregard it. But what if a similar sensory event were to happen all the time, with yellow chair images momentarily appearing and disappearing without a trace? Would I continue to perceive and disregard them? Peirce suggests, in an interesting discussion of the action of optical illusions on the mind over time, that insofar as the yellow chair percepts were *regularly* recognized as illusory, they would become much less vivid and possibly disappear altogether:

> It is one of the recognized difficulties of all psycho-physical measurement that the faculties rapidly become educated to an extraordinary degree. Thus, contrast-colors, when properly exhibited, are incredibly vivid. One is not easily persuaded that they are not real. Yet the experimenter becomes in time almost incapable of perceiving them. This is a case in which the same educational course which gives control over appearances which sometimes do and sometimes do not accord with the mass of experiences, only serves to strengthen the forcefulness of those appearances which always do so accord (CP 7.647).

The contrast-colour illusion involves staring at a bright patch of colour (e.g. red) then looking at a white surface, which will initially appear to be the opposite color to the one stared at (e.g. green), but over time, as the mind learns that the white surface is 'not really green' the perceived greenness literally fades. These very obvious illusions enable the training of the percept-to-perceptual judgment relation (which it was noted above largely takes place in childhood) to be resuscitated and studied within an observable time-period. The most important thing to note is that this training of perception is a *rational* process. It proceeds by the mind making the best overall sense it can of ponecipuum, percipuum and antecipuum as a total package. If that involves reinterpreting something just apparently seen as in fact illusory, then so be it. Hookway puts this point well:

> What we experience is not just a clash between our beliefs and our experience; we often experience incoherence within the experience itself, which simultaneously involves anticipations and thwarts those very anticipations. The fact that, in these cases, 'the perceptual judgment, and the percept itself, seems to keep shifting from one general aspect to another and back again (CP 5.183) shows that the percept is not 'entirely free from…characters that are proper to interpretations (CP 5.184) (HOOKWAY, 2012, p. 17).

Thus future experience can, at least in part, *literally determine previous experience.* Although it was earlier offered as an obvious homily that "experience just shows us what is right in front of us", in Peirce's understanding of perception "nothing at all […] is absolutely confrontitional".

5. Peirce on Perceiving Necessity: "The ideas […] cluster in spite of our will"

So how does all this work in the case of perceiving mathematical necessity? What is a *mathematical* percept? It is obviously not going to be quite like the percept of a yellow chair. What is it going to be like? Let us take seriously Peirce's repeated claims that mathematics is *as experimental a science as physics*:

> I have sometimes been tempted to think that mathematics differed from an ordinary inductive science hardly at all except for the circumstance that experimentation which in the positive sciences is so costly in money, time, and energy, is in mathematics performed with such facility that the highest inductive certainty is attained almost in the twinkling of an eye (PEIRCE, *Pragmatism as a Principle and Method of Right Thinking*, p.131).

The laboratory equipment of the mathematician is the *diagram*. Let us reconsider *Fig. 1*, which demonstrates the necessary truth of $2 \times 3 = 3 \times 2$, and re-examine it using Peirce's theory of perception.

5.1. The Mathematical Percept

The percept for *Fig. 1* is literally impossible to describe in words, as are all percepts. However I will try to convey indirectly something of my phenomenology while 'getting' this proof.

The diagram presents two rows of three stars (2×3), and at the same time three columns of two stars (3×2). As I understood the proof, I had a sudden perception of the horizontal and vertical arrangements of the stars *as one*. It was as if the same 5 ovals were 'holding together' both arrangements, although this is only a metaphor as the arrangements are not strictly *parts* of the diagram. Pace Hume, the arrangements are *abstractable but not separable*, since one cannot reproduce the first arrangement without reproducing the second, and vice versa. Such is true of many objects in mathematics, and to this degree Hume's *Separate Imaginability Criterion of Distinctness* would appear to beg the question against the claim that we might perceive necessary relationships between mathematical objects, by failing to accord them object status at all.

At the same time, looking at the diagram and thinking about other possible arrangements that might be abstracted from it, such as (3×3) or (3×4), I perceived some kind of primitive blocking of those other options. It is as if I could *feel myself not being able to think of* them.[20] We might call this primitive blocking or constraint, in homage to Wittgenstein, 'the hardness of the mathematical must'.[21] Thus Peirce writes: "Although mathematics deals with ideas and not the world of sensory experience, its discoveries are not arbitrary dreams but something to which our minds are forced […] (MOORE, 2010, p. 41)".

5.2. The Mathematical Perceptual Judgment

My past mathematical training in concepts such as number and multiplication meant that I represented the primitive blockage of certain thoughts felt by my mind when viewing the diagram by means of the proposition $2 \times 3 = 3 \times 2$. This proposition is a general symbol which can be put to an indefinite number of further uses, such as food rationing, or bathroom tile design. Or it may be integrated into higher-level arithmetic theory—for example in framing propositions such as "multiplication is commutative".

5.3. The Mathematical Percipuum

Having formulated the proposition $2 \times 3 = 3 \times 2$, I cannot help interpreting the blockage of certain alternative constructions felt by my mind when viewing the diagram to mean not only that the proposition is true, but that 2 is the *only* option for filling the missing place in the equation $2 \times 3 = 3 \times \_$. As a modal epistemology, this might sound 'too easy'. One might ask: in positing this mysterious percept-to-

20 The reader is at this point invited to try this experiment for herself.

21 I have explored related terminology in earlier publications, e.g. LEGG, 2012.

perceptual-judgment interpretative relationship, which with the right training tells us what to think in mathematics, what has been gained?

What has been gained is an honesty about what actually happens when we work out knowledge of what Hume called 'relations of ideas'. The lack of rational justification, the blind compulsion to believe, is greater in our mathematical even than in our sensory experience, because the former is shot through so intimately with necessary truth. (This is what mathematical diagrams are for, to distil and isolate necessary from contingent features of reality.) In that sense, our lack of understanding of why and how we are forbidden to think in certain ways is even greater in mathematics than anywhere else. As Peirce puts it:

> We can know nothing about the percept,—but only experience it in its totality,—except through the perceptual judgment, and this likewise compels acceptance without any assignable reason. This indefensible compulsiveness of the perceptual judgment is precisely what constitutes the cogency of mathematical demonstration (CP 7.659).

From understanding this, hopefully we may derive a measure of epistemic humility about the nature of the powers which mastery of mathematics, the alleged jewel in the crown of rationalism, really endows us with as thinkers.

6. Occult-hood Revisited

Hume assumed that naturalists must avoid positing occult powers at all costs, on pain of falling into epistemic laziness and even idiocy. Peirce on the other hand calmly evaluates the phenomena, admits that certain powers of the mind *are occult* and—characteristically—pragmatically clarifies the notion of occult-hood so that it can do real philosophical work:

> [The clustering of ideas] is either due to an outward occult power or to an inward one. That it is due to some occult power is plain from this, that the ideas although they are in our own minds and thus normally subject to our will, cluster in spite of our will, and that in certain regular ways. This is a sound argument that some power not ourselves does that which ordinarily we ourselves do. But it is occult in this sense, that nothing more about it can be learned by mere observation of these phenomena (MOORE, 2010, p. 50).

The above quote comes from a fascinating piece from 1895 where Peirce tries his hand at explicating his category of Thirdness purely in terms of the clustering of ideas into ever more general "sets". Here he directly addresses—and appears to be trying to recapitulate—Hume's theory of the association of ideas according to the three "qualities" of *Resemblance*, *Contiguity* and *Cause and Effect*. To Hume's idea that this associationist theory might serve as a kind of total 'physics of the mind'—beyond which a true naturalist should "restrain the intemperate desire of searching into causes" (*Treatise*, 1, I, v, p. 13)—Peirce has a few things to say. He suggests that the Principle of Contiguity does not replace an occult power but enshrines one (in what Peirce calls the 'outer world'):

> Contiguity *consists* in ideas being brought together in experience, and is not the *cause* of it. That cause is that occult power acting like our wills, though with far greater might, which lies behind experience, and which the old philosophers called Nature […] (MOORE, 2010, p. 50).

Similarly, the Principle of Resemblance does not replace an occult power but enshrines one (in what Peirce calls the 'inner world'):

> […] as before, it is to be remarked that the resemblance *consists* in the ideas clustering together (as scarlet and crimson, insist upon clustering together), and is not the *cause* of the clustering. That cause is an occult power which seems to lie behind the inward world just as Nature lies behind the outward world. It is often called *Reason* (MOORE, 2010, p. 51).

Relatedly, Peirce's final verdict on telepathy in 1902, having studied the matter carefully, is that it is only "somewhat more remote from perception than the conjectures by which physicists so often hit on the truth" (CP 7.681).

Conclusion

Hume has been very influential in establishing a dualism in contemporary philosophy between on the one hand matters of fact which are part of reality but contain no necessity, and on the other hand relations of ideas which contain necessity but are not part of reality. But mathematics is part of reality insofar as our ideas about it are *forced*. Despite analytic philosophers' bafflement as to how it should be possible, we do perceive necessary truths. Perception is in fact the *only* way in which we gain knowledge of necessity, insofar as all necessary reasoning involves experimenting upon diagrams. Although many of these diagrams may be understood to represent ideal objects, by 'ideal' is meant not 'unreal' but merely that they distil broad-scale necessary features of reality in a convenient viewing package. Thus our supposed perennial empiricist insight, that "experience just shows us what is right in front of us", is just not true—at least if "what is right in front of us" is understood in the sense in which Hume did, namely actual, concrete objects, and nothing more.

Peirce's nuanced theory of perception allows us to see that Hume is interestingly right and wrong phenomenologically about perceiving necessity. We might say that Hume is *correct* in his devastating analysis of causal necessity that there is no necessary truth in his immediate experience, if we understand that to mean his *percept*. He is just incorrect that it follows from this that he is perceiving no such thing in reality—i.e. that he has no *perceptual judgment* of it. Hume failed to see this because he insisted on viewing *ideas as copies of impressions*. This was arguably a phenomenological defect in his philosophising.[22] Hume also failed (at

22 Peirce addressed this defect explicitly in a discussion of colour-memory. He criticized Hume for claiming that when we remember we perceive the same idea, only with less 'force' and 'vivacity' by stating, "If this were a correct statement of the difference, we should remember [an object] being less red than it is; whereas, in fact, we remember the color with very great precision for a few moments […] although we do not see anything like it. We carry away absolutely nothing of the color except the *consciousness that we could recognize it*" (CP 5.300). Once again this is a description of an index.

least in this arena) to see that there are other kinds of representations than copies. That an idea might *index* an impression rather than resembling it could have been an interesting idea for Hume's naturalistic theory of mind, except that the indexing process as conceived by Peirce requires to be scaffolded by general habits which Hume's nominalism has no room for.

Our final moral is that although mathematics seems the most rational of all sciences—clear and distinct as Descartes put it—ironically, from another view ('looking inside our heads') it is one of the most mysterious of all:

> The action of Nature is a wonder to us; but that of Reason is not usually so…We seem to comprehend Reason. We flatter ourselves that we grasp its very *noumenon*. But it is really as occult as Nature. It is only because its effects are for the most part familiar to us from infancy that they are not surprising (MOORE, 2010, p. 51).

References

ALMEDER, R. "Peirce's Theory of Perception". *Transactions of the Charles S. Peirce Society*, vol. 6, nº 2, pp. 99-110, 1970.

AYER, A. J. *The Problem of Knowledge*. London: Macmillan, 1956.

BLACKBURN, S. "Morals and Modals". In: G. MacDONALD, G and WRIGHT, C. (eds.). *Fact, Science and Morality*: Essays on A.J. Ayer's *Language, Truth and Logic*. Oxford: Blackwell, 1986, pp. 119-141.

BONJOUR, L. "Epistemological Problems of Perception". *Stanford Encyclopedia of Philosophy*, 2007. Available at: http://plato.stanford.edu/entries/perception-episprob/. Downloaded: 20 Feb, 2014.

BRAUDE, S.E. "Peirce on the Paranormal". *Transactions of the Charles S. Peirce Society*, vol. 34, nº 1, pp. 203-224, 1998.

BROWN, D. *The Lost Symbol*. New York: Doubleday, 2009.

BROWN, J.R. *Philosophy of Mathematics*: An introduction to the world of proofs and pictures. London and New York: Routledge, 1999.

CHURCH, J. "Seeing Reasons". *Philosophy and Phenomenological Research*, vol. 80, nº 3, pp. 638-670, 2010.

CRANE, T. "The Problem of Perception". *Stanford Encyclopedia of Philosophy*, 2011. Available at: http://plato.stanford.edu/entries/perception-problem/. Downloaded: 22 Feb, 2014.

DAVIDSON, D. "Mental Events". In: *Essays on Actions and Events*. Oxford: Clarendon Press, pp. 207-27, 1980 [orig. 1970].

DEROSSET, L. "Production and Necessity". *The Philosophical Review*, vol. 118, nº 2, pp. 153-181, 2009.

ELLIS, B. *Scientific Essentialism*. Cambridge: Cambridge University Press, 2001.

EINSTEIN, A. *Ideas and Opinions*. New York: Crown Publishers, 1954.

FLEW, A. "Infinite Divisibility in Hume's Treatise". In: *Hume: A re-evaluation*. LIVINGSTON, D and KING, J. (eds.). New York: Fordham University Press, 1976.

FOGELIN, R. *Hume's Skepticism in the Treatise of Human Nature*. London: Routledge and Kegan Paul, 1985.

FORSTER, P. *Peirce and the Threat of Nominalism*. Cambridge: Cambridge University Press, 2011.

FRANKLIN, J. "Achievements and Fallacies in Hume's Account of Infinite Divisibility". *Hume Studies*, vol. 20, nº 1, pp. 85-102, 1994.

GIAQUINTO, M. *Visual Thinking in Mathematics*. Oxford: Oxford University Press, 2007.

JACKSON, F. and PRICE, H. "Naturalism and the Fate of the M-Worlds". *Proceedings of the Aristotelian Society, Supp. Vol*., vol. 71, nº 1, pp. 269-282, 1997.

HALE, B. "The Source of Necessity". *Philosophical Perspectives*, vol. 16, *Language and Mind*, pp. 299-319, 2002.

HOOKWAY, C. *The Pragmatic Maxim*: Essays on Peirce and pragmatism. Oxford: Oxford University Press, 2012.

_____."'… a sort of composite photograph': Pragmatism, ideas, and schematism". *Transactions of the Charles S. Peirce Society*, vol. 38, nº 1/2, pp. 29-45, 2002.

HUEMER, M. "Sense-Data". *Stanford Encyclopedia of Philosophy*, 2011. Available at: http://plato.stanford.edu/entries/sense-data/. Downloaded: 27 March, 2014.

HUME, D. *A Treatise of Human Nature*. SELBY-BIGGE, L. A. (ed.). Oxford: Oxford University Press, 1978 [1739-40].

_____. *Enquiry Concerning Human Understanding*. Oxford: Oxford University Press, 1975 [1748].

KIM, Jaegwon "Causation, Nomic Subsumption, and the Concept of Event". *Journal of Philosophy*, vol. 70, nº 8, pp. 217-36, 1973.

LEGG, C. "The Problem of the Essential Icon". *American Philosophical Quarterly*, vol. 45, nº 3, pp. 207-232, 2008.

_____. "The Hardness of the Iconic Must: Can Peirce's Existential Graphs assist modal epistemology?" *Philosophia Mathematica,* vol. 20, nº1, pp. 1-24, 2012.

_____. "What is a Logical Diagram?". In: *Visual Reasoning with Diagrams*. MOKTEFI, A., SHIN, S.-J. (eds.). Dordrecht: Springer, pp. 1-18, 2013.

LEWIS, D. 1973. "Causation". *The Journal of Philosophy*, vol. 70, no. 17, pp. 556-567, 1973.

_____. *On the Plurality of Worlds*. Oxford: Blackwell, 1986.

_____. "Truthmaking and Difference-Making". *Noûs*, vol. 35, pp. 602-615, 2001.

LOWE, E.J. 2012. "What is the Source of Our Knowledge of Modal Truths?" *Mind*, vol. 121, nº 484, pp. 919-950, 2012.

MANDERS, K. "The Euclidean Diagram". In: *The Philosophy of Mathematical Practice*. Mancosu (ed.). Oxford: Oxford University Press, 2008.

MCDOWELL, J. *Mind and World*. Cambridge, Mass.: Harvard University Press, 1994.

MENZIES, P., & PRICE, H. "Causation as a Secondary Quality". *British Journal for the Philosophy of Science*, vol. 44, nº 2, pp. 187-203, 1993.

MUMMA, J. "Proofs, Pictures and Euclid". *Synthese*, vol. 175, nº 2, pp. 255-287, 2010.

ODERBERG, D.S. "Hume, the Occult, and the Substance of the School". *Metaphysica*, vol. 13, no. 2, pp. 155-174, 2009.

PEIRCE, C.S. *Essential Peirce, vol. 1*: Selected philosophical writings (1867-1893). HOUSER, N. and KLOESEL, C. (eds.). Indianapolis: Indiana University Press, 1992.

_____. *Essential Peirce, vol. 2*: Selected philosophical writings (1893-1913). HOUSER, N. and KLOESEL, C. (ed.). Indianapolis: Indiana University Press, 1998.

_____. *Collected Papers of Charles Sanders Peirce*. HARTSHORE, C. and WEISS, P. (eds.). Massachusetts: Harvard University Press, 1931-1958.

_____. *Philosophy of Mathematics*: Selected writings. MOORE, M. E. (ed.). Bloomington: Indiana University Press, 2010.

_____. *Pragmatism as a Principle and Method of Right Thinking*: The 1903 Harvard Lectures on Pragmatism. TURRISI, P. (ed.). Albany: SUNY Press, 1997.

POPKIN, R.H. "David Hume: His Pyrrhonism and His Critique of Pyrrhonism". *The Philosophical Quarterly*, vol. 1, nº 5, pp. 385-407, 1951.

ROSENTHAL, S. "The Percipuum and the Issue of Foundations", 2001. Available at: http://www.digitalpeirce.fee.unicamp.br/perros.htm. Downloaded: 26 October, 2012.

_____. "Peirce's Theory of Perceptual Judgment: An Ambiguity". *Journal for the History of Philosophy*, vol. 7, nº 3, pp. 303-314, 1969.

RUSSELL, B. *The Problems of Philosophy*. New York: Oxford University Press, 1912.

SELLARS, W. "Sensa or Sensings: Reflections on the Ontology of Perception". *Philosophical Studies*, vol. 41, nº 1, pp. 83-111, 1982.

SHERRY, D. "The Role of Diagrams in Mathematical Arguments". *Foundations of Science*, vol. 14, nº 1-2, pp. 59-74, 2009.

WILSON, J. "What is Hume's Dictum, and Why Believe It?" *Philosophy and Phenomenological Research*, vol. 80, nº 3, pp. 595-637, 2010.

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